General Game Playing
Statistical Search

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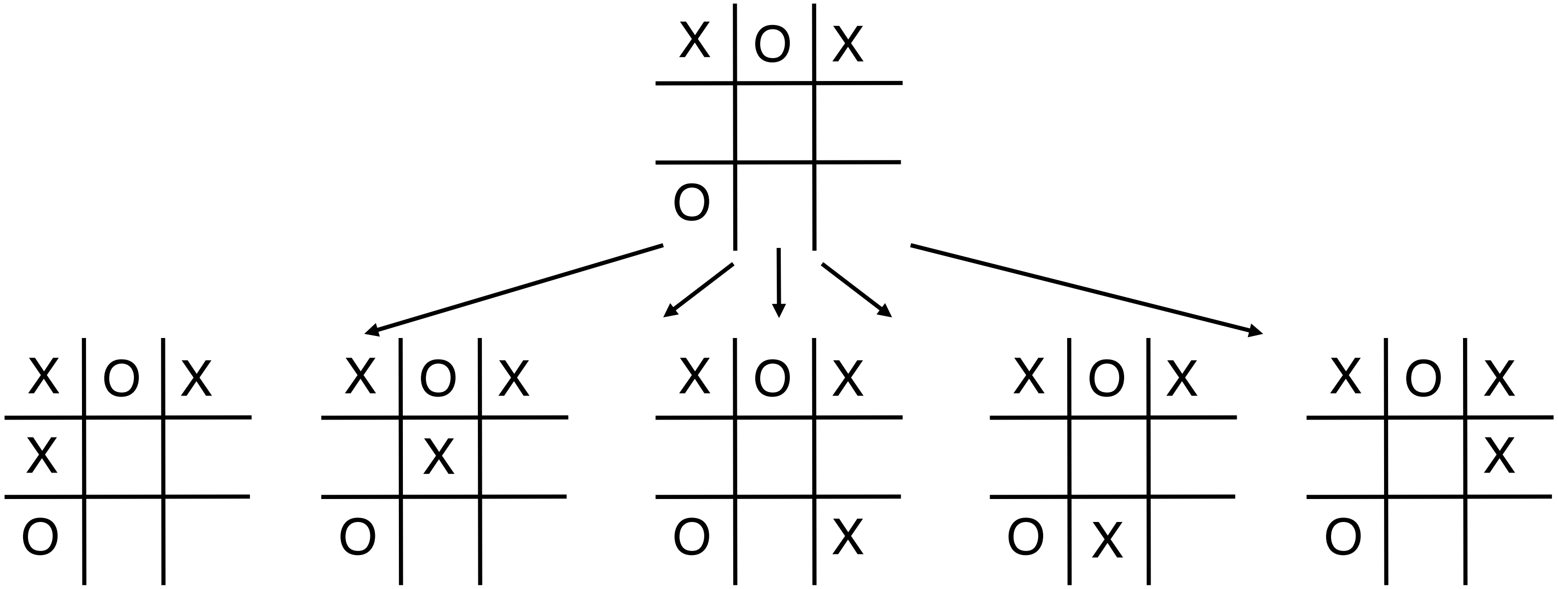
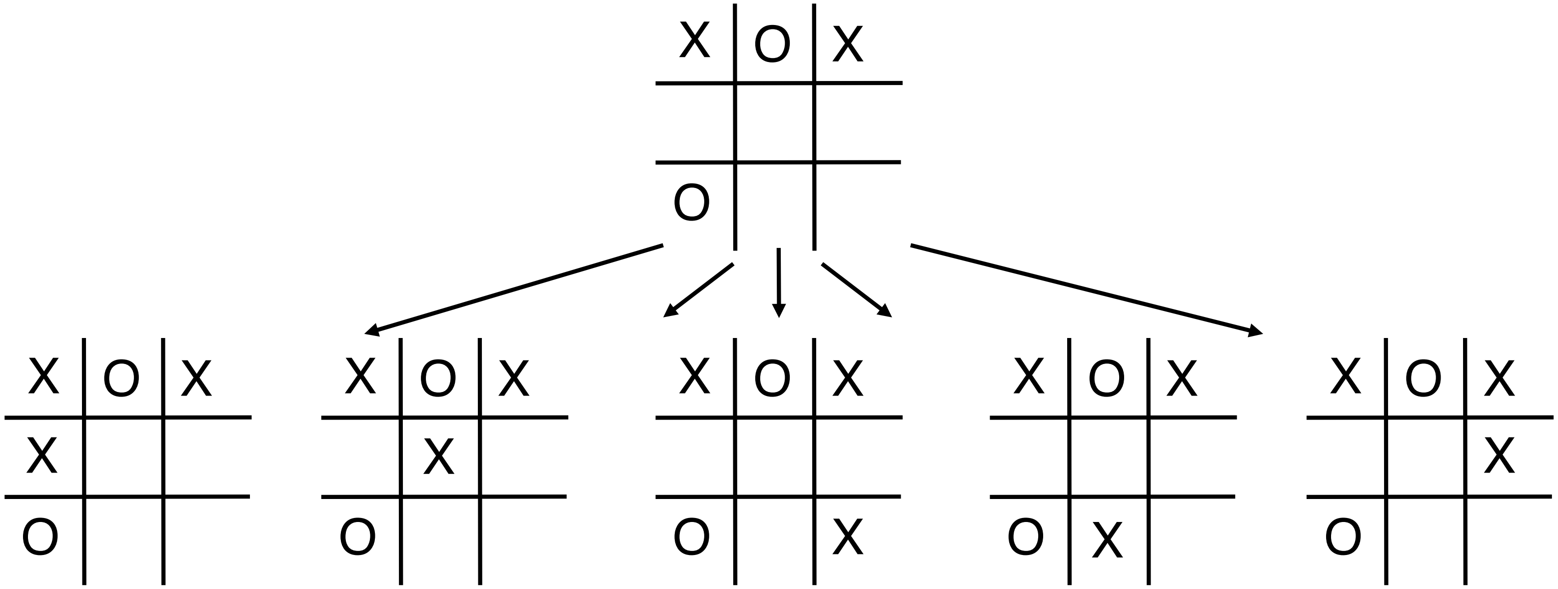
Incomplete Search

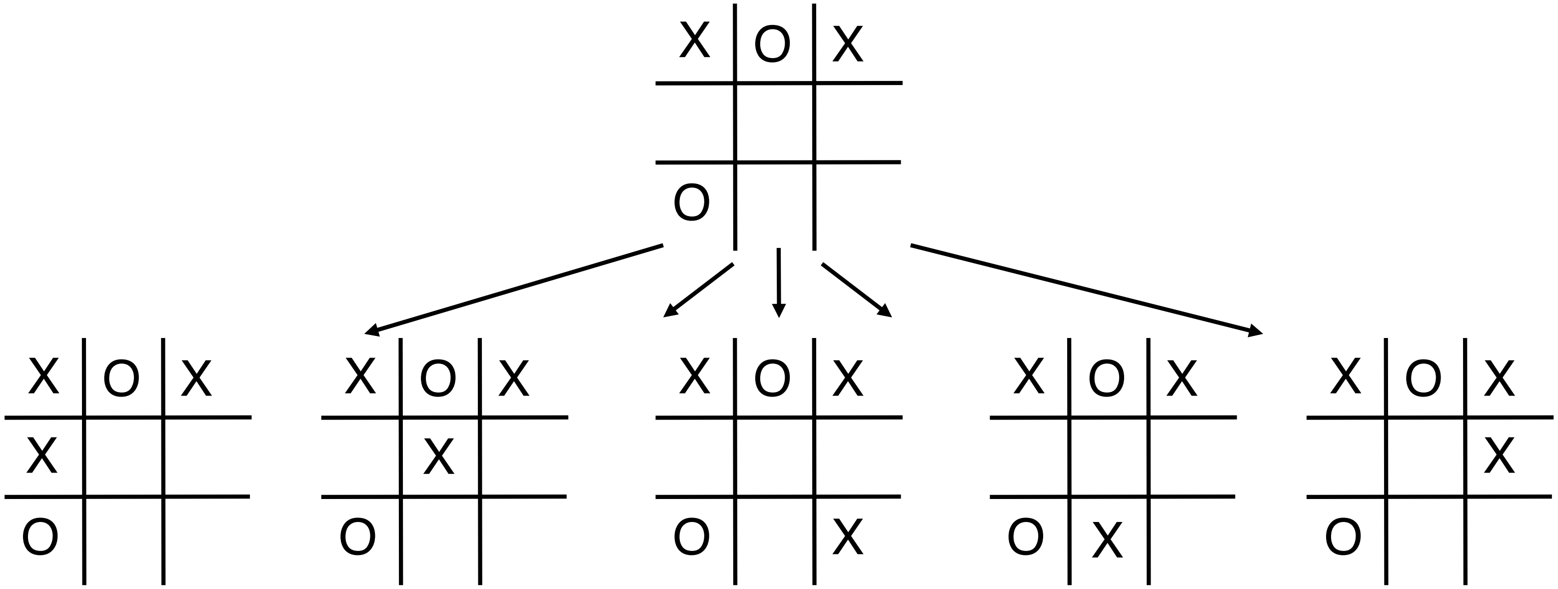


Evaluation of Non-Terminal States



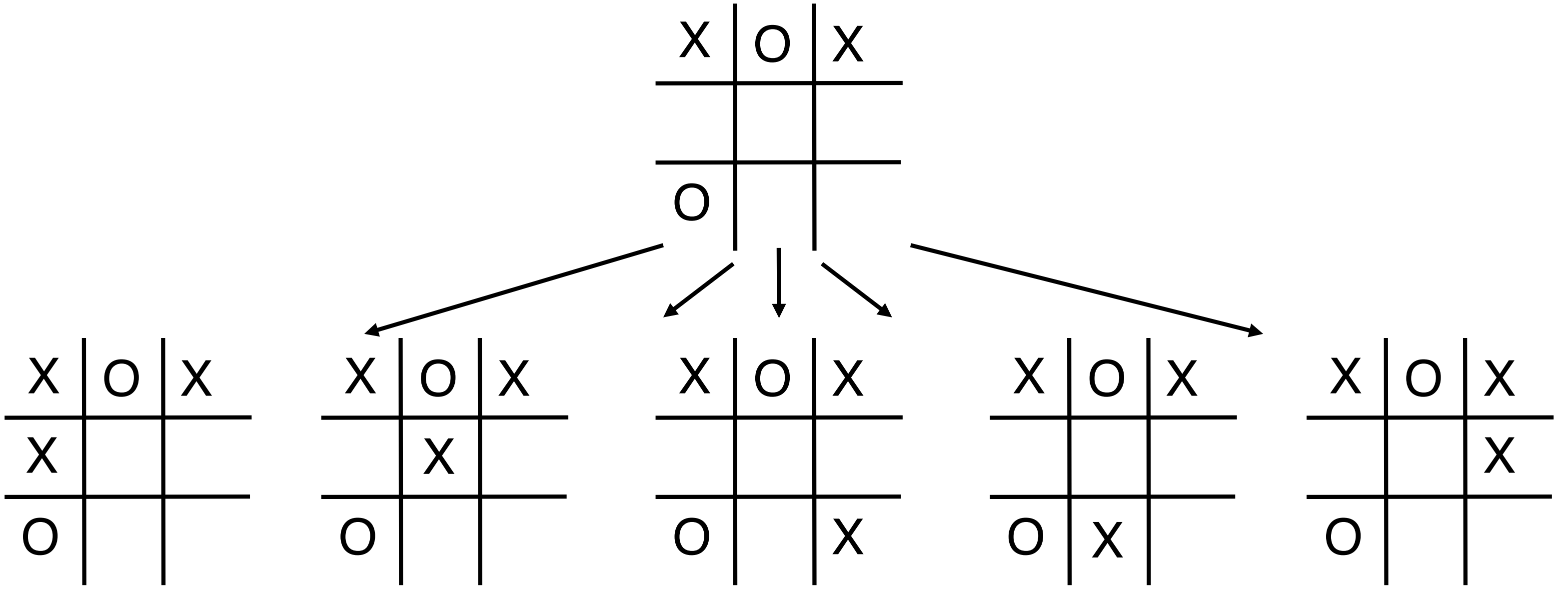
How do we evaluate non-terminal states? ✓

Choice of Depth



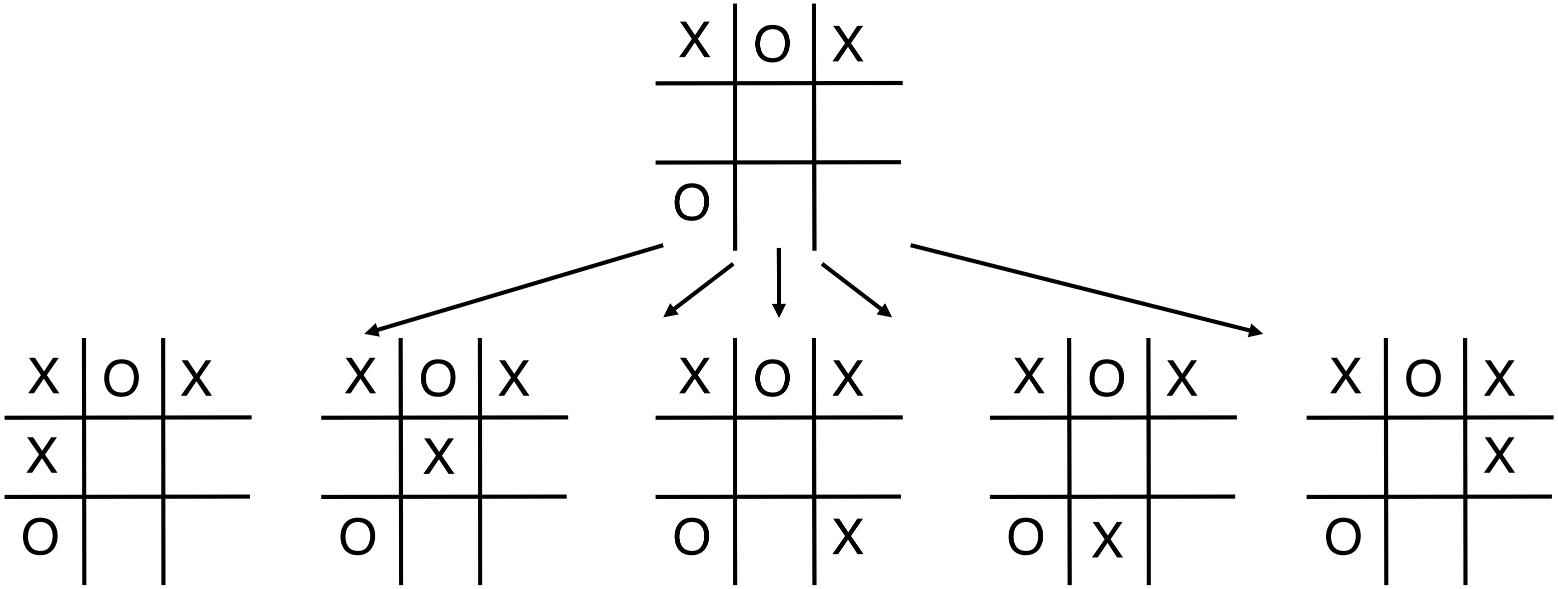
To what depth should we search? ✓

Variable Depth Search



Can we search different branches to different depths?

Persistence



Can we preserve tree across moves or phases of ID?

Greedy

Overview

Create a game tree data structure.

Select branch of tree, sub-branch, etc. till reach fringe.
Expand fringe node.
Propagate values back to the root.

Repeat until time runs out.
Select move w/ best estimated value.

Replace game tree with subtree as each move is made

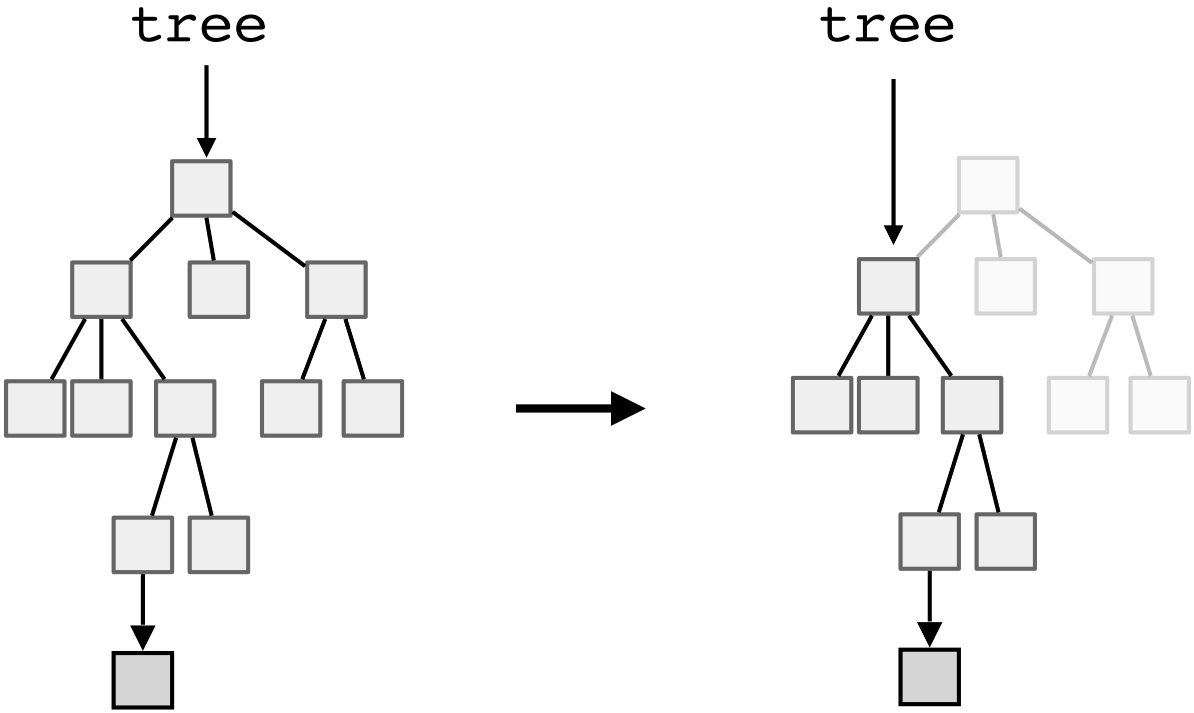
start

```
var role, roles, state, library, startclock, playclock;
var tree = {};

function start (r,rs,sc,pc)
 {role = r;
  library = definemorerules([],rs.slice(1));
  roles = findroles(library);
  state = findinits(library);
  startclock = parseInt(sc);
  playclock = parseInt(pc);
  var reward = parseInt(findreward(role,state,library));
  tree = makenode(state,findcontrol(state,library),reward);
  return 'ready'}

function makenode (state,mover,reward)
 {return {state:state,
          actions:[],
          children:[],
          mover:mover,
          utility:reward}}
```

play

```
function play (move)
 {if (move!==nil)
     {tree = subtree(move,tree); state = tree.state};
  if (findcontrol(state,library)!==role) {return false};
  var deadline = Date.now()+(playclock-2)*1000;
  while (Date.now()<deadline) {process(tree)};
  return selectaction(tree)}
```

Updating the Tree After Move



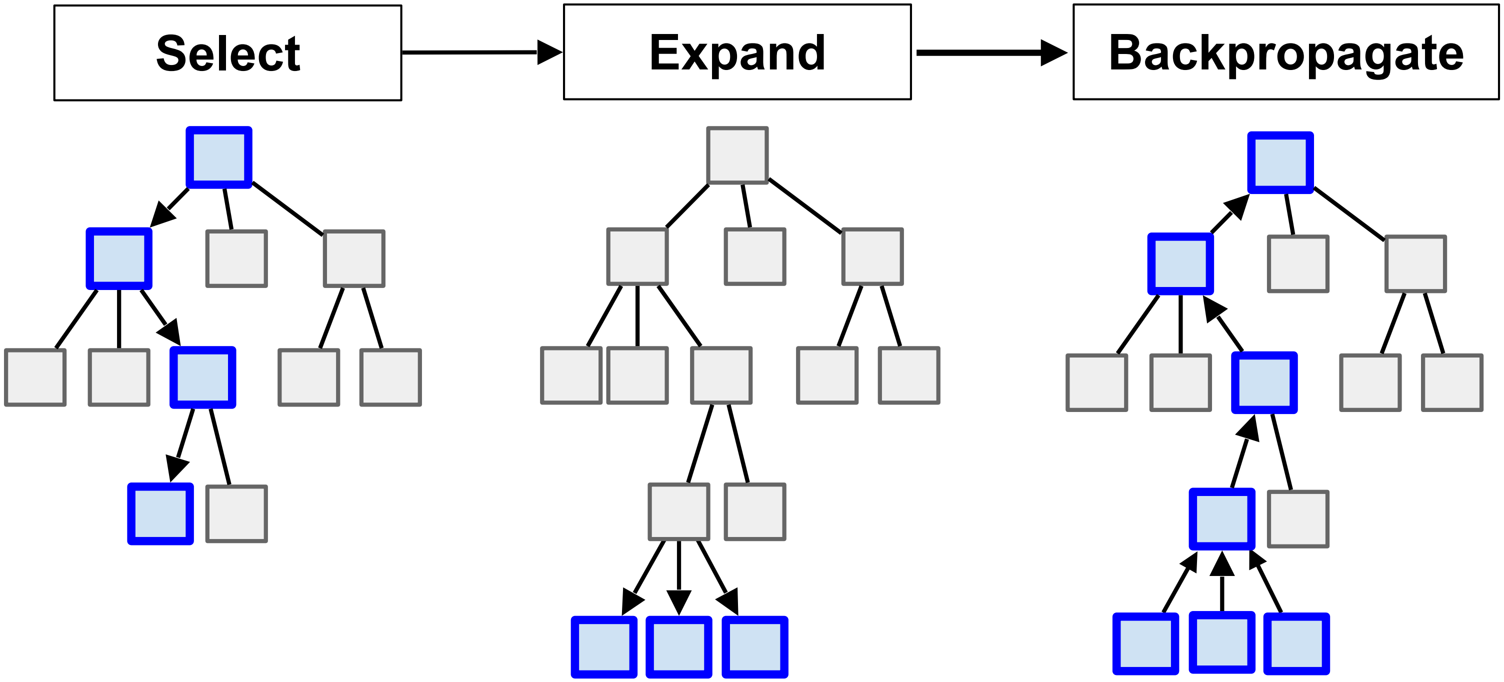
subtree

```
function subtree (move,node)
 {if (node.children.length===0) {expand(node)};
  for (var i=0; i<node.actions.length; i++)
      {if (equalp(move,node.actions[i]))
          {return node.children[i]}}
  return node}
```

play

```
function play (move)
 {if (move!==nil)
     {tree = subtree(move,tree); state = tree.state};
  if (findcontrol(state,library)!==role) {return false};
  var deadline = Date.now()+(playclock-2)*1000;
  while (Date.now()<deadline) {process(tree)};
  return selectaction(tree)}
```

Processing the Tree



Select

Expand

Backpropagate

process

```
function process (node)
 {if (findterminalp(node.state,library)) {return true};
  if (node.children.length===0) {expand(node)}
     else {process(select(node))};
  update(node);
  return true}
```

Selection

Expansion

Backpropagation

select

```
function select (node)
 {var total = node.visits;
  var child = node.children[0];
  var score = child.utility;
  for (var i=1; i<node.children.length; i++)
      {var newchild = node.children[i];
       var newscore = newchild.utility;
       if (newscore>score)
          {child = newchild; score = newscore}};
  return child}
```

expand

```
function expand (node)
 {node.actions = findlegals(node.state,library);
  for (var i=0; i<node.actions.length; i++)
      {var newstate = simulate(node.actions[i],node.state,library);
       var newmover = findcontrol(newstate,library);
       var newscore = parseInt(findreward(role,newstate,library));
       node.children[i] = makenode(newstate,newmover,newscore)};
  return true}

function makenode (state,mover,reward)
 {return {state:state,
          actions:[],
          children:[],
          mover:mover,
          utility:reward}}
```

update

```
function update (node)
 {if (node.mover===role) {node.utility = scoremax(node)}
     else {node.utility = scoremin(node)};
 return true}

function scoremax (node)
 {var score = node.children[0].utility;
  for (var i=1; i<node.children.length; i++)
      {var newscore = node.children[i].utility;
       if (newscore>score) {score = newscore}};
  return newscore}

function scoremin (node)
 {var score = node.children[0].utility;
  for (var i=1; i<node.children.length; i++)
      {var newscore = node.children[i].utility;
       if (newscore<score) {score = newscore}};
  return newscore}
```

play

```
function play (move)
 {if (move!==nil)
     {tree = subtree(move,tree); state = tree.state};
  if (findcontrol(state,library)!==role) {return false};
  var deadline = Date.now()+(playclock-2)*1000;
  while (Date.now()<deadline) {process(tree)};
  return selectaction(tree)}
```

selectaction

```
function selectaction (node)
 {var action = node.actions[0];
  var score = node.children[0].utility;
  for (var i=1; i<node.children.length; i++)
      {var newscore = node.children[i].utility;
       if (newscore>score)
          {action = node.actions[i]; score = newscore}};
  return action}
```

Selection Functions

Utility

```
function select (node)
 {var total = node.visits;
  var child = node.children[0];
  var score = child.utility;
  for (var i=1; i<node.children.length; i++)
      {var newchild = node.children[i];
       var newscore = newchild.utility;
       if (newscore>score)
          {child = newchild; score = newscore}};
  return child}
```

Visits

```
function makenode (state,mover,reward)
 {return {state:state, actions:[], children:[], mover:mover,
          utility:reward, visits:0}}

function update (node)
 {if (node.mover===role) {node.utility = scoremax(node)}
     else {node.utility = scoremin(node)};
  node.visits = node.visits+1;
  return true}

function selectnode (node)
 {var child = node.children[0];
  var visits = node.children[0].visits;
  for (var i=1; i<node.children.length; i++)
      {var newvisits = node.children[i].visits;
       if (newvisits<visits)
          {child = node.children[i]; visits = newvisits}};
  return child}
```

Exploitation + Exploration

```
function select (node)
 {var total = node.visits;
  var child = node.children[0];
  var score = value(child.utility,child.visits,total);
  for (var i=1; i<node.children.length; i++)
      {var newchild = node.children[i];
       var newvalue = newchild.utility;
       var newvisits = newchild.visits;
       var newscore = value(newvalue,newvisits,total);
       if (newscore>score)
          {child = newchild; score = newscore}};
  return child}

function value (utility,visits,total)
 {var score = (utility + Math.round((1 - visits/total)*100));
  return score}
```

Exploitation *Exploration*

Ideas

Some players use different formulas for combing Exploitation and Exploration.

$$\text{selectValue} = \frac{\text{node.utility}}{\text{node.visits}} + C \times \sqrt{\frac{\ln(\text{node.parent.visits})}{\text{node.visits}}}$$

Some players find value in recording the **standard deviation** of scores at each node or where the **change in estimated values** and favor expanding those nodes with the highest value to clarify the uncertainty.

Problem

The preceding implementation does not distinguish between values that are *guaranteed* (because the player searched to the end of the tree) from those that are *estimated* (based on heuristic evaluation function).

Fix by adding additional information to each node.

Monte Carlo Tree Search

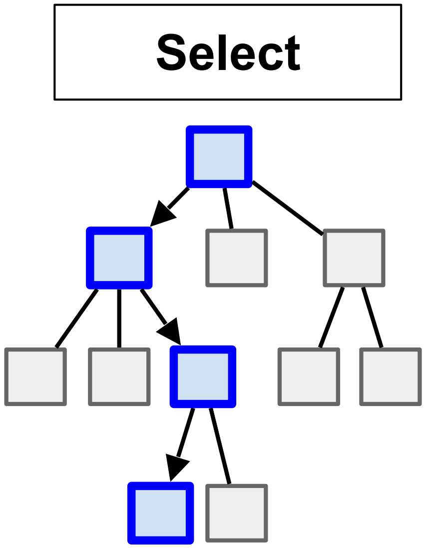
Basic Idea

Monte Carlo Tree Search (MCTS) is a search method that relies on random probes to estimate state values. Blend of Greedy and MCS.

Like MCS:
  Builds up game tree incrementally
  Random probes to end of game to estimate state values

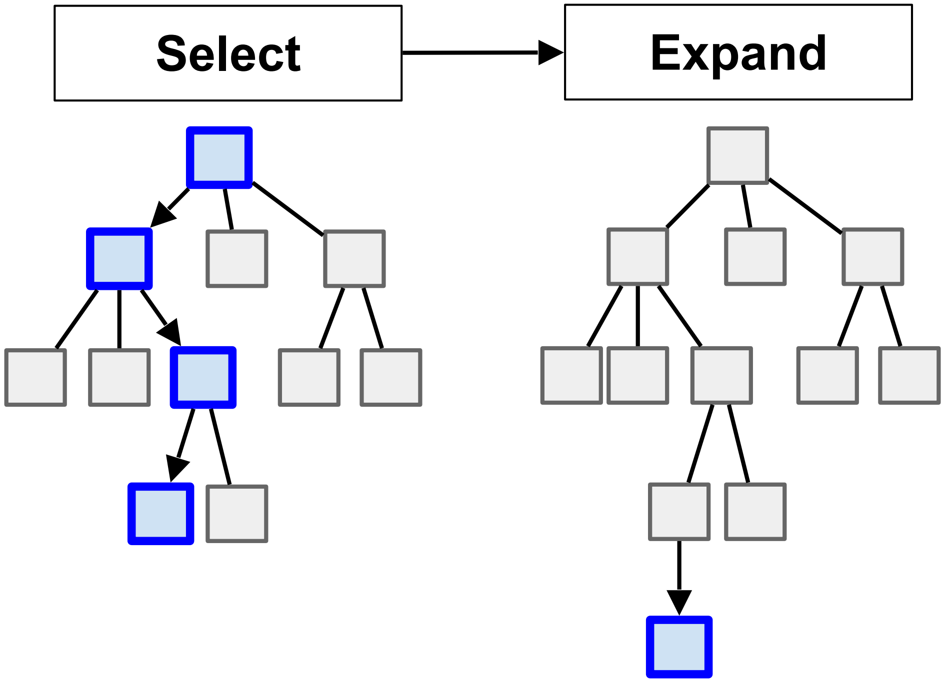
Like Greedy:
  MCTS expands non-uniformly
  Uses combination of Exploitation and Exploration

Overview



Select

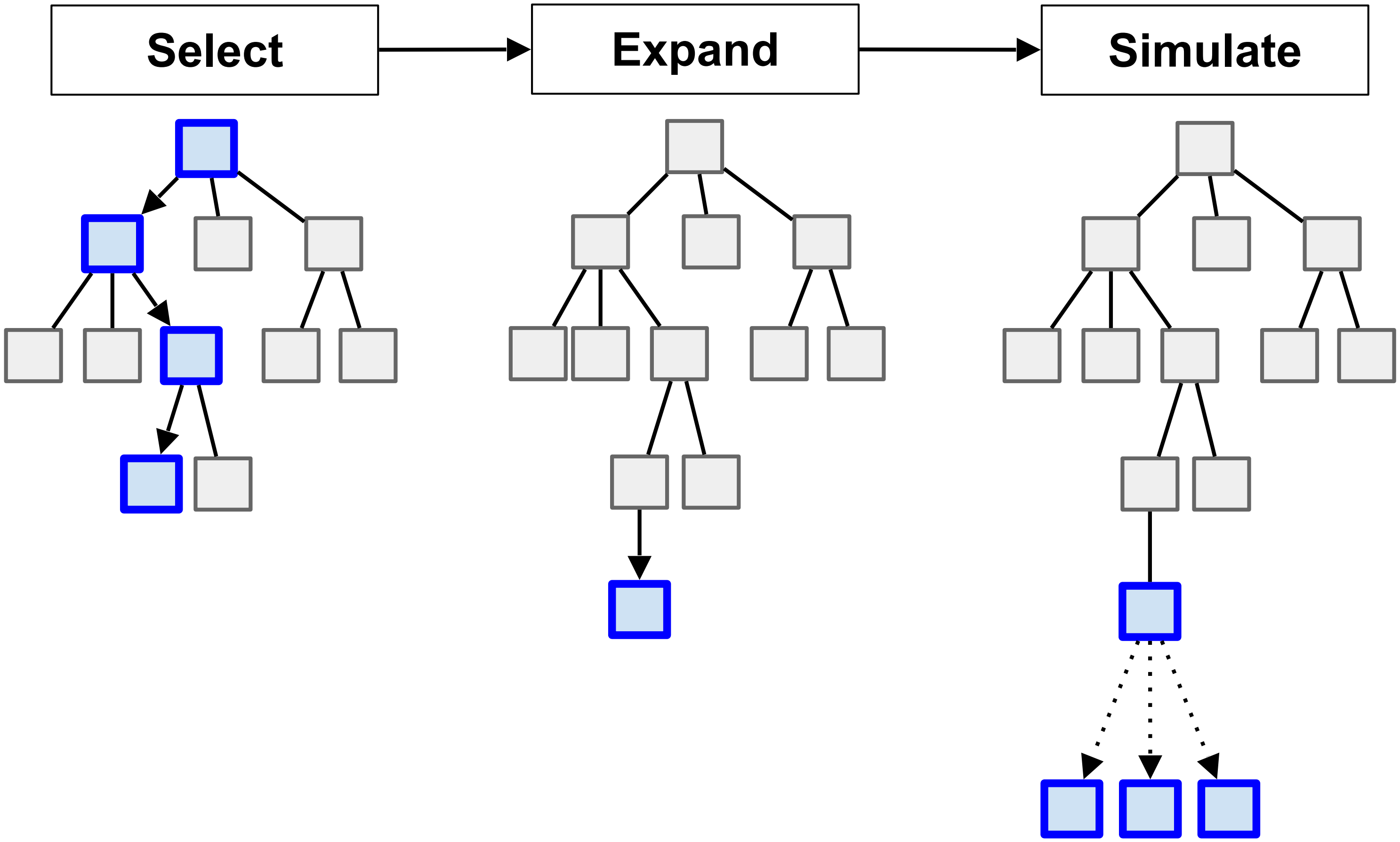
Overview



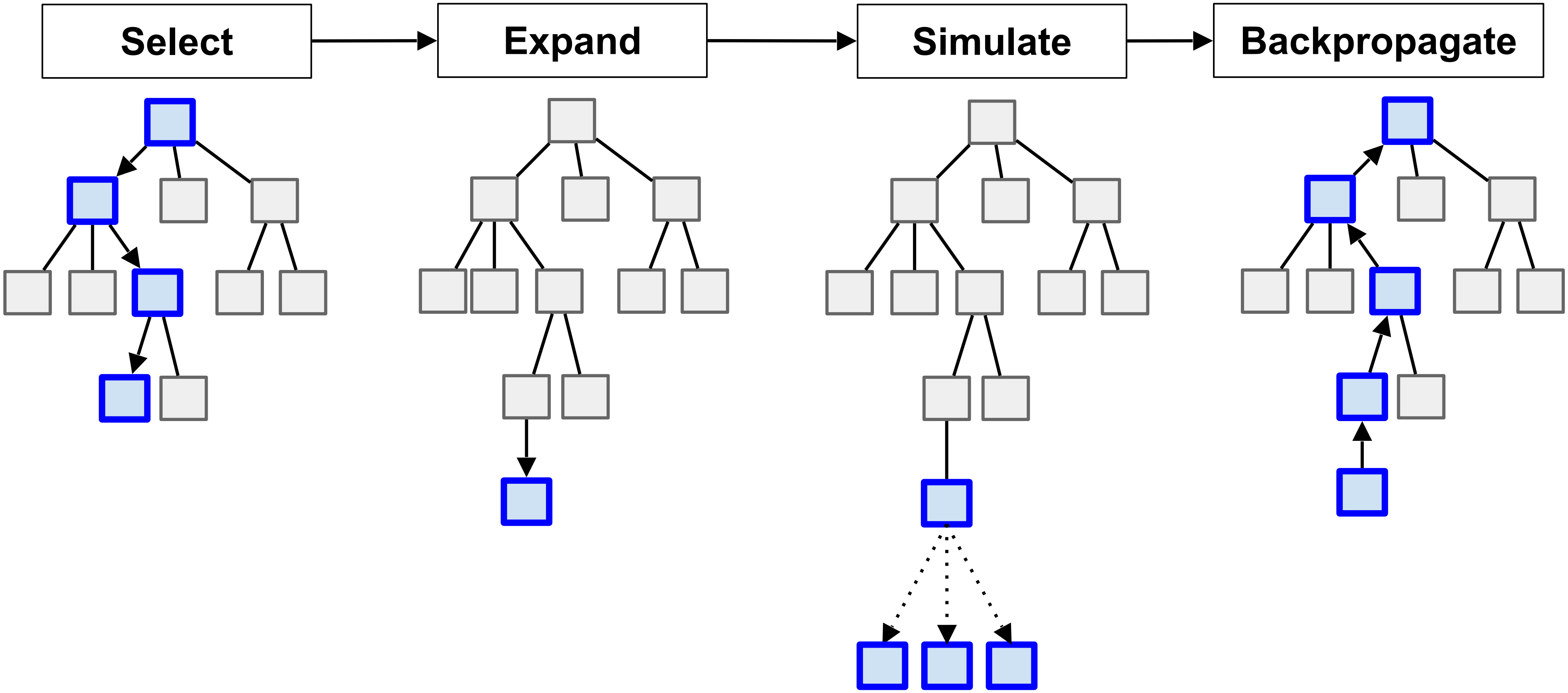
Select

Expand

Overview

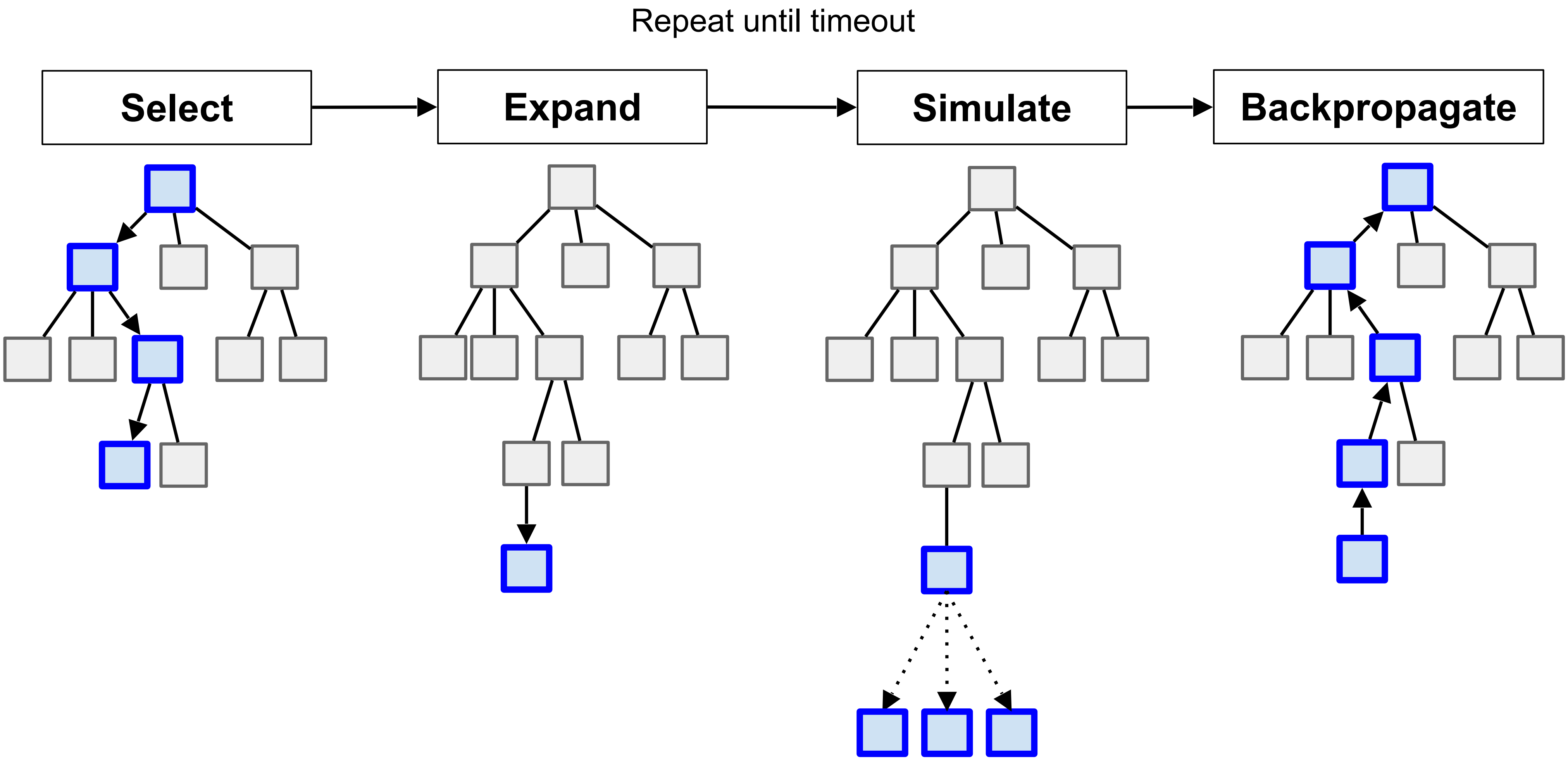


Overview



Select → Expand → Simulate → Backpropagate

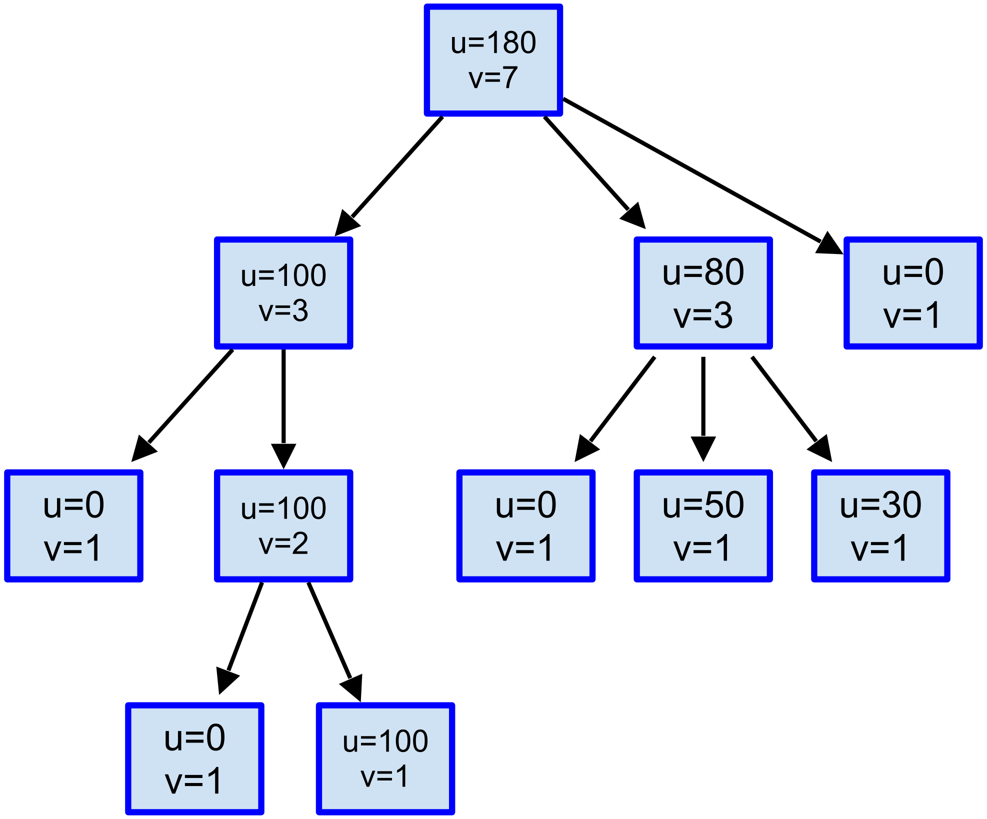
Overview



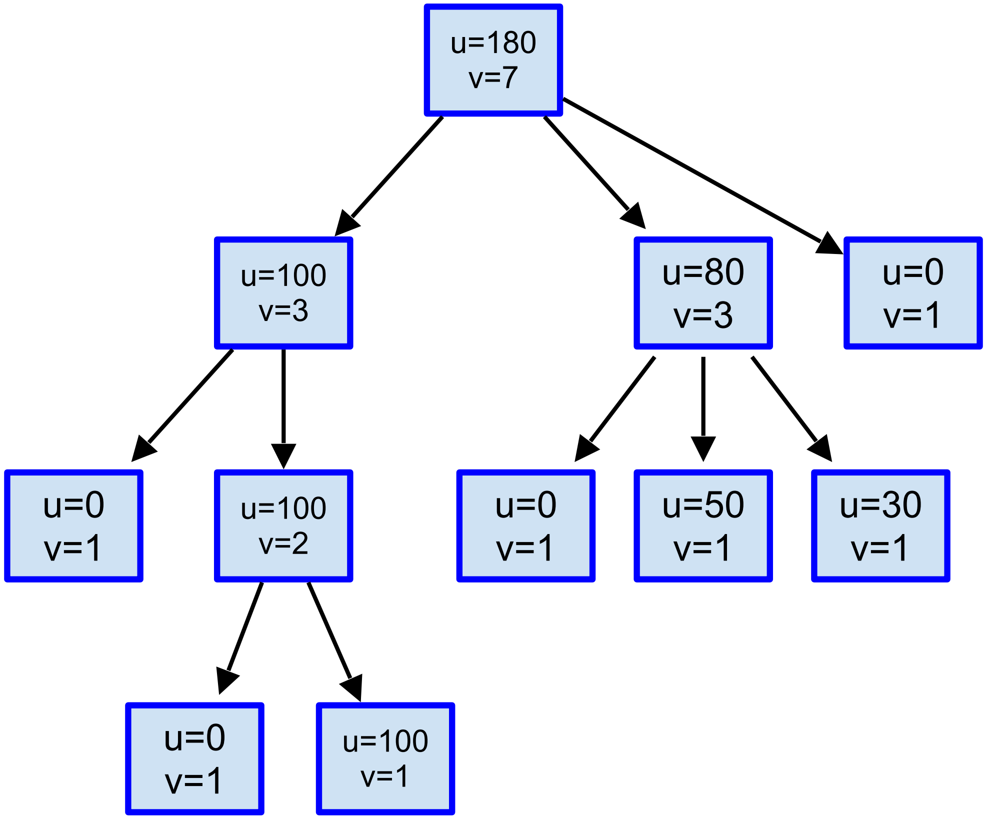
Repeat until timeout

Select → **Expand** → **Simulate** → **Backpropagate**

Single Player



Select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v=7
u=100
v=3
u=80
v=3
u=0
v=1
u=0
v=1
u=100
v=2
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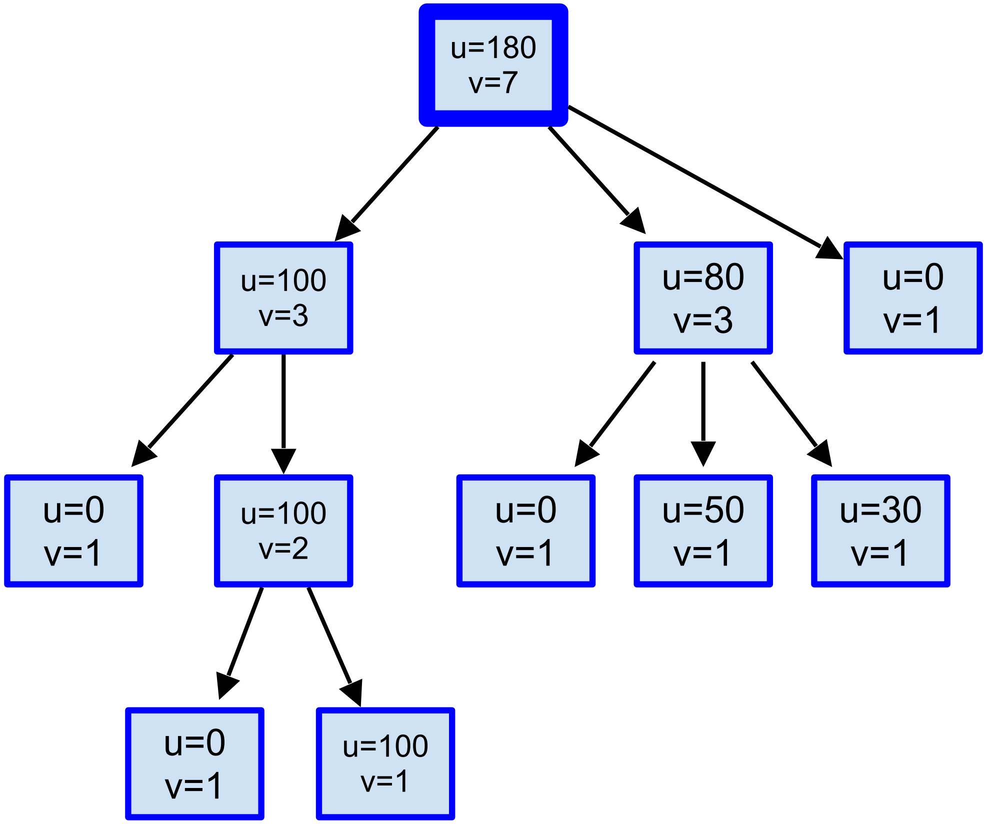
u=0
v=1
u=100
v=1

Select


u=180
v=7
u=100
v=3
u=80
v=3
u=0
v=1
u=0
v=1
u=100
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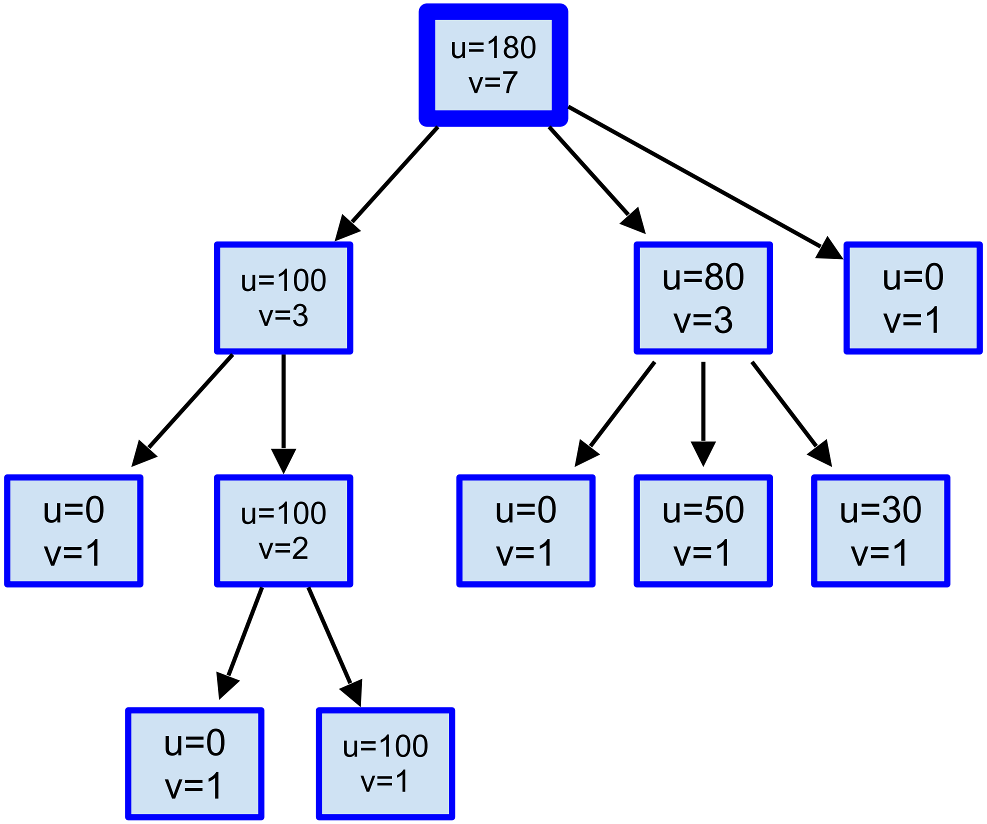
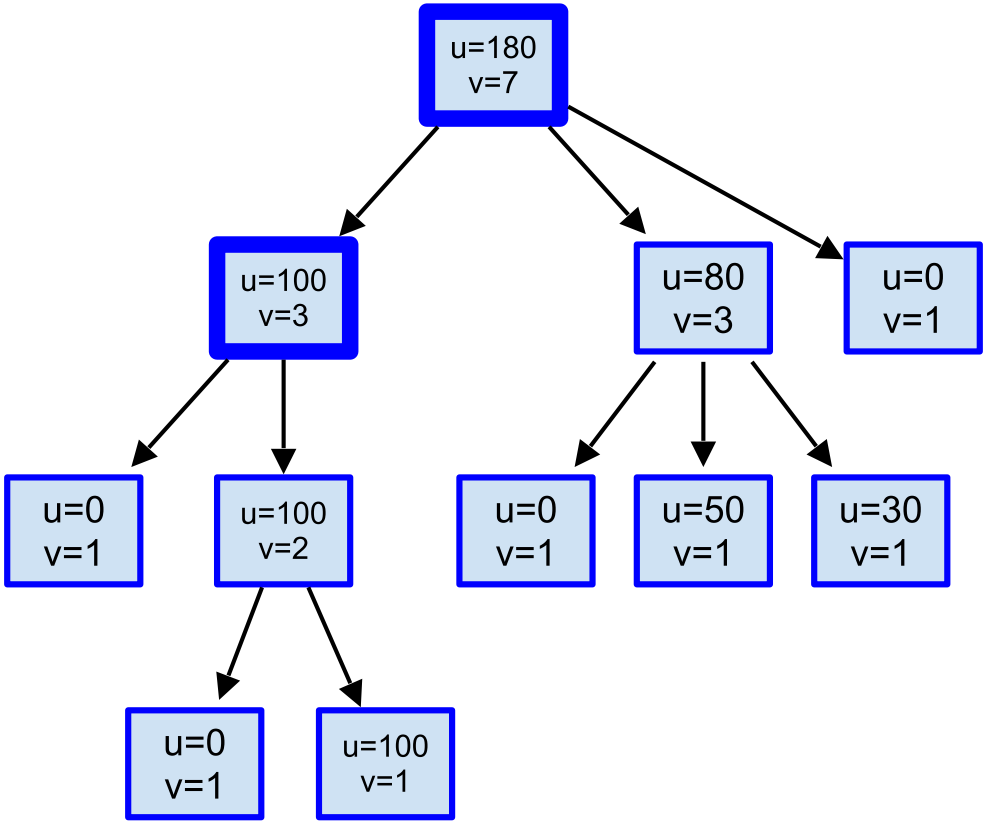
v=1
u=50
v=1
u=30
v=1
u=0
v=1
u=100
v=1

Select

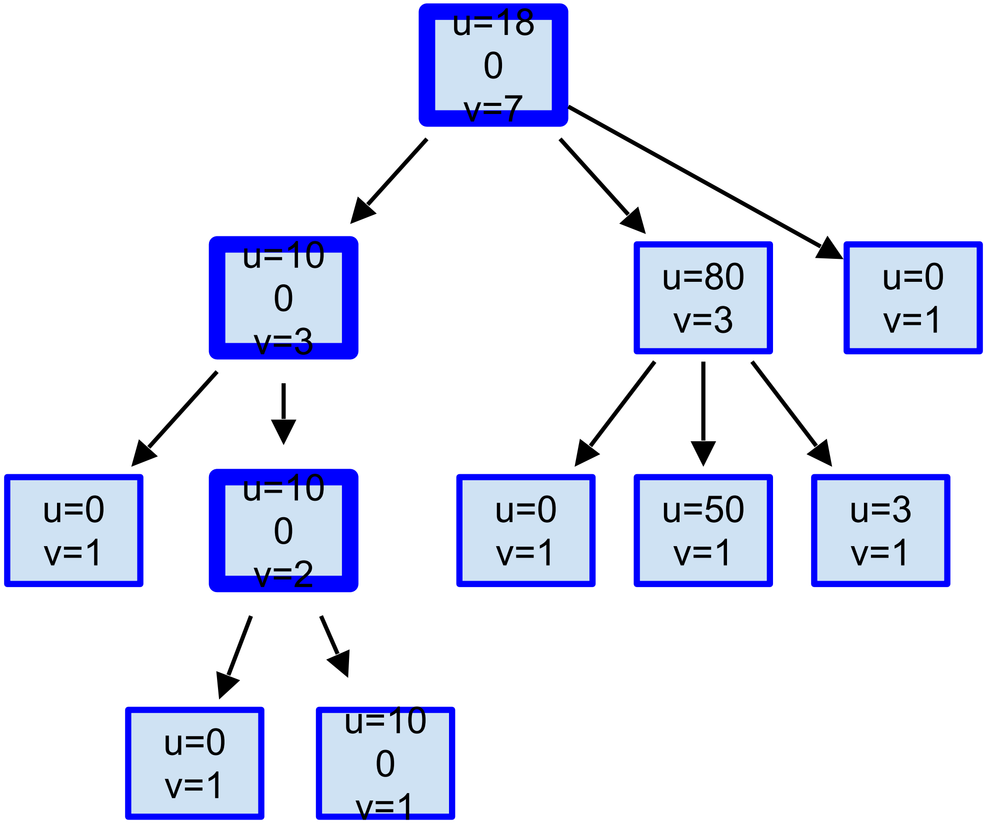


$$\text{selectValue} = \frac{\text{node.utility}}{\text{node.visits}} + C \times \sqrt{\frac{\ln(\text{node.parent.visits})}{\text{node.visits}}}$$

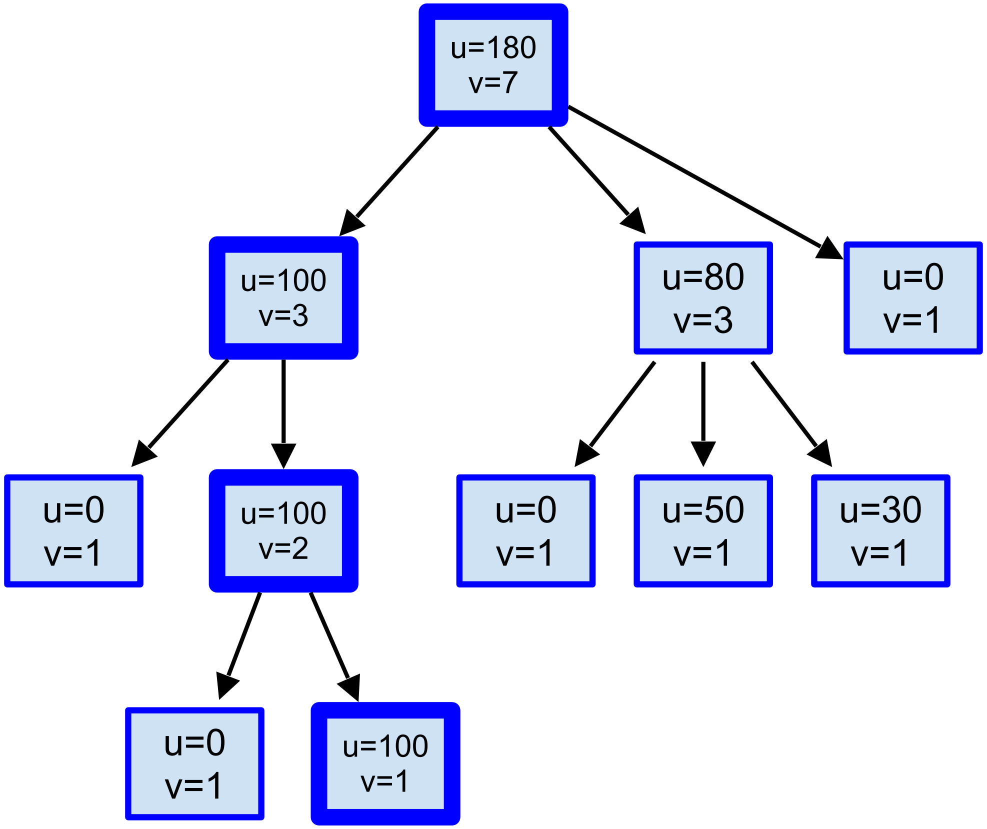
Select


u=180
v=7
u=100
v=3
u=80
v=3
u=0
v=1
u=0
v=1
u=100
v=2
u=0
v=1
u=50
v=1
u=30
v=1
u=0
v=1
u=100
v=1

Select


u=18
0
v=7
u=10
0
v=3
u=80
v=3
u=0
v=1
u=0
v=1
u=10
0
v=2
u=0
v=1
u=50
v=1
u=3
v=1
u=0
v=1
u=10
0
v=1

Select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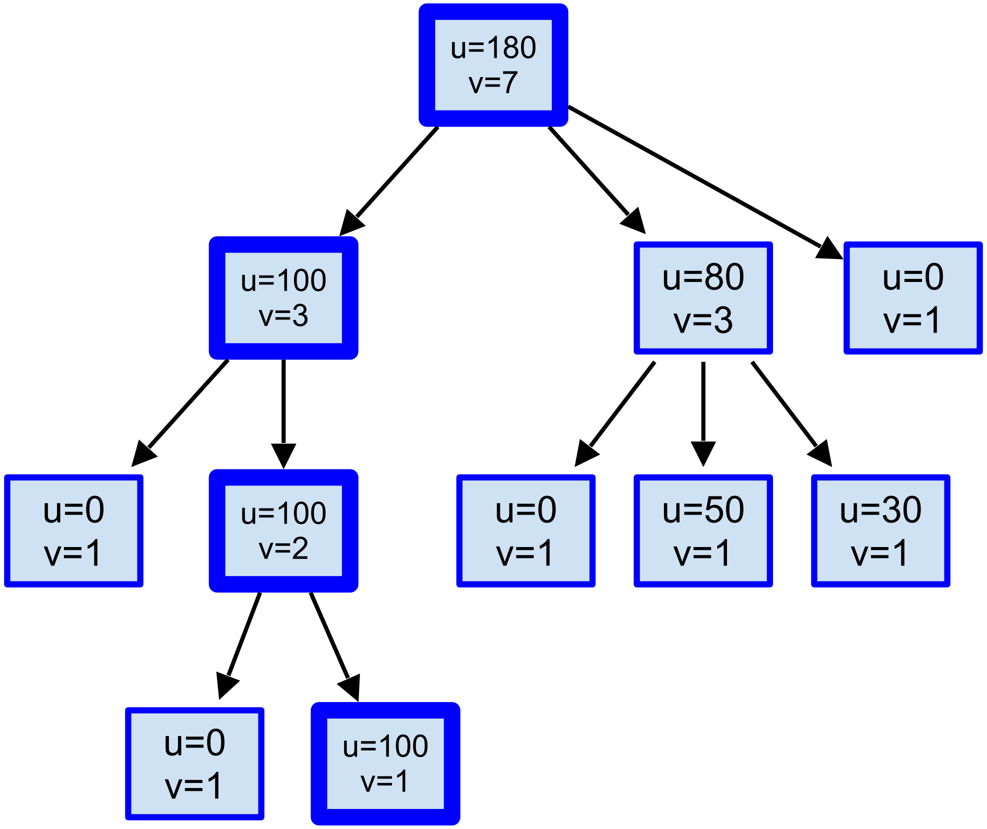
v=7
u=100
v=3
u=80
v=3
u=0
v=1
u=0
v=1
u=100
v=2
u=0
v=1
u=50
v=1
u=30
v=1
u=0
v=1
u=100
v=1

Select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v=7
u=100
v=3
u=80
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v=1
u=0
v=1
u=100
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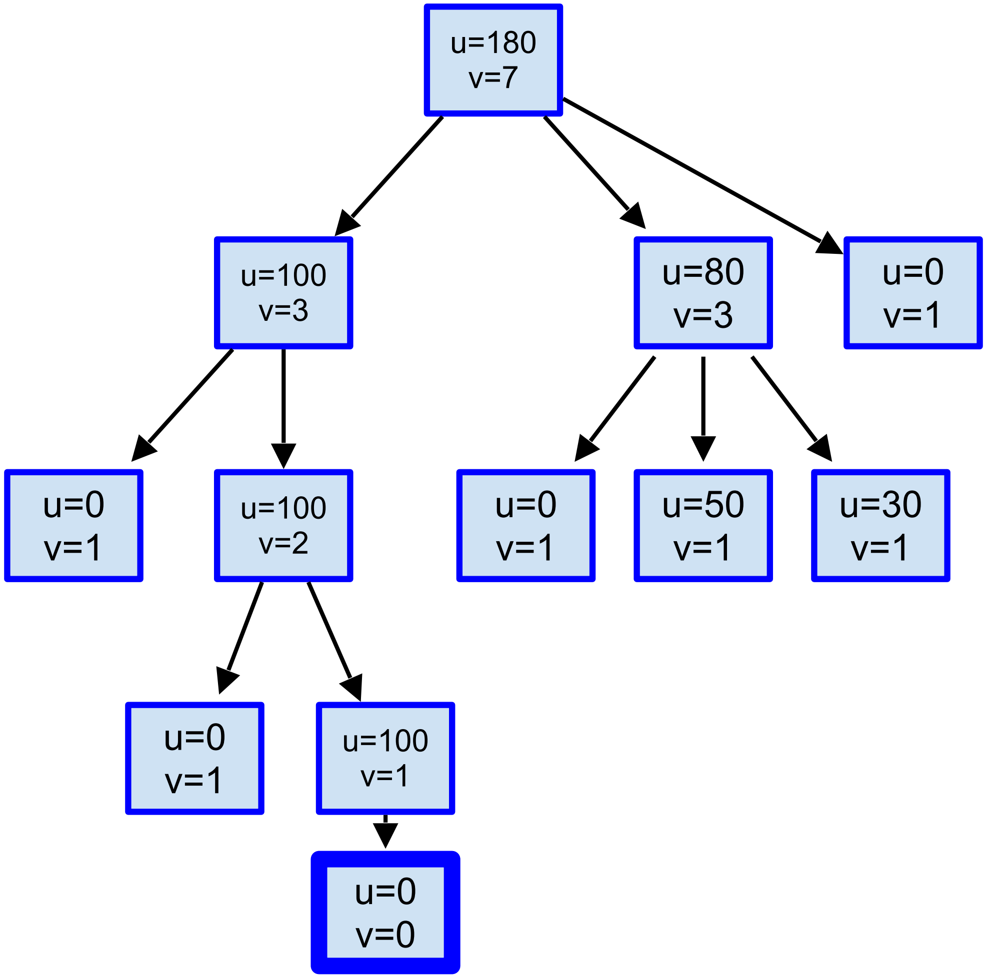
v=1
u=50
v=1
u=30
v=1
u=0
v=1
u=100
v=1

Expand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v=7
u=100
v=3
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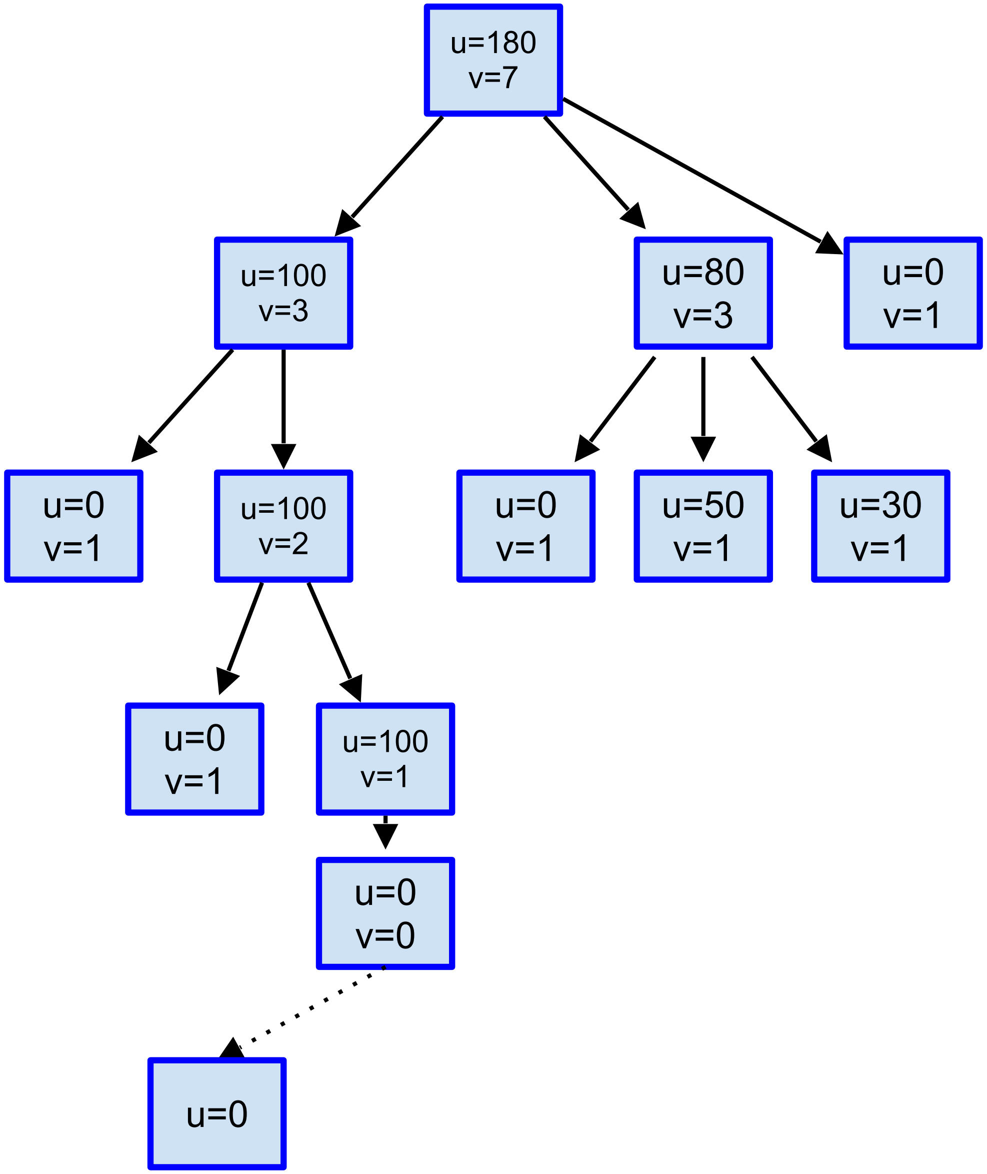
v=3
u=0
v=1
u=0
v=1
u=100
v=2
u=0
v=1
u=50
v=1
u=30
v=1
u=0
v=1
u=100
v=1
u=0
v=0

Simulate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u=100
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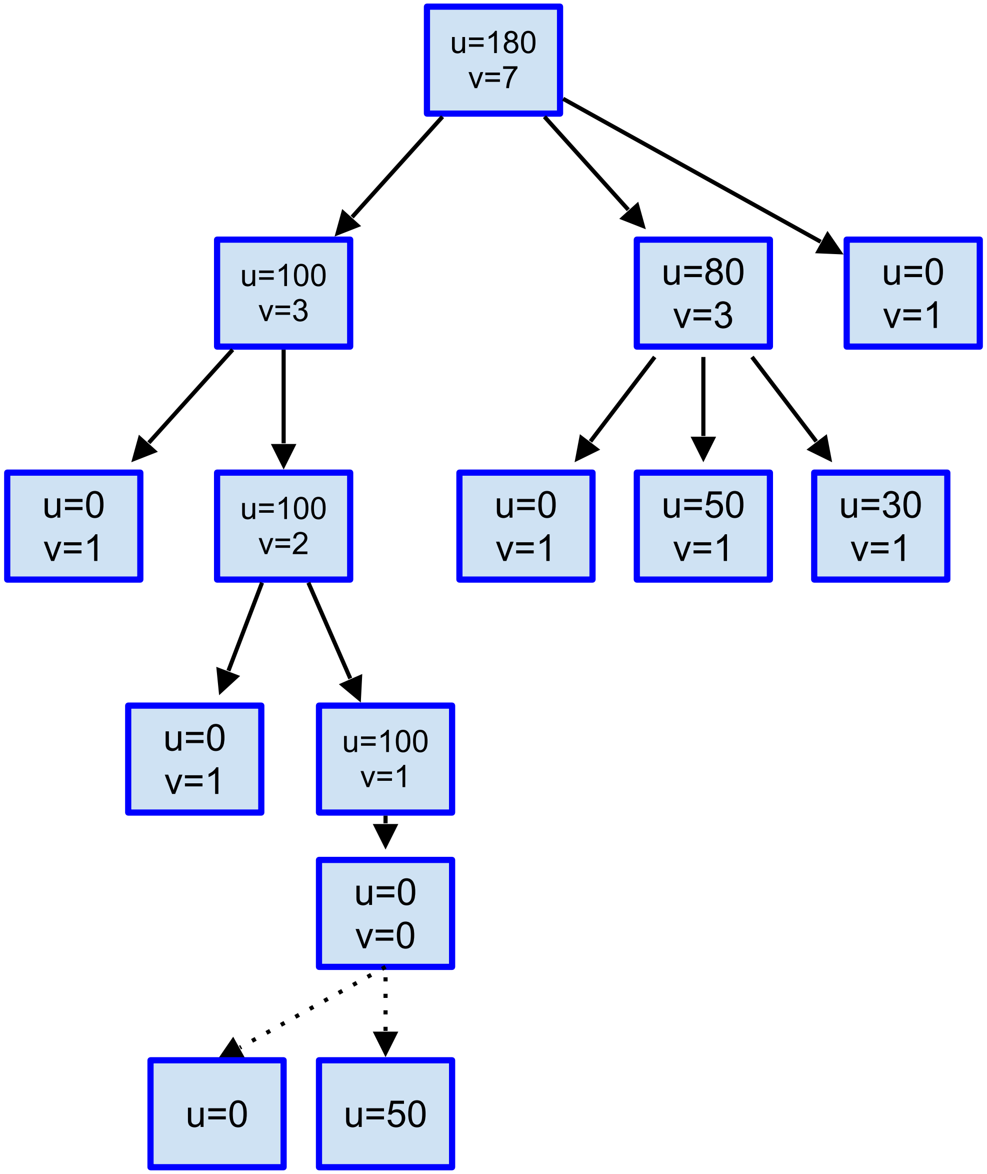
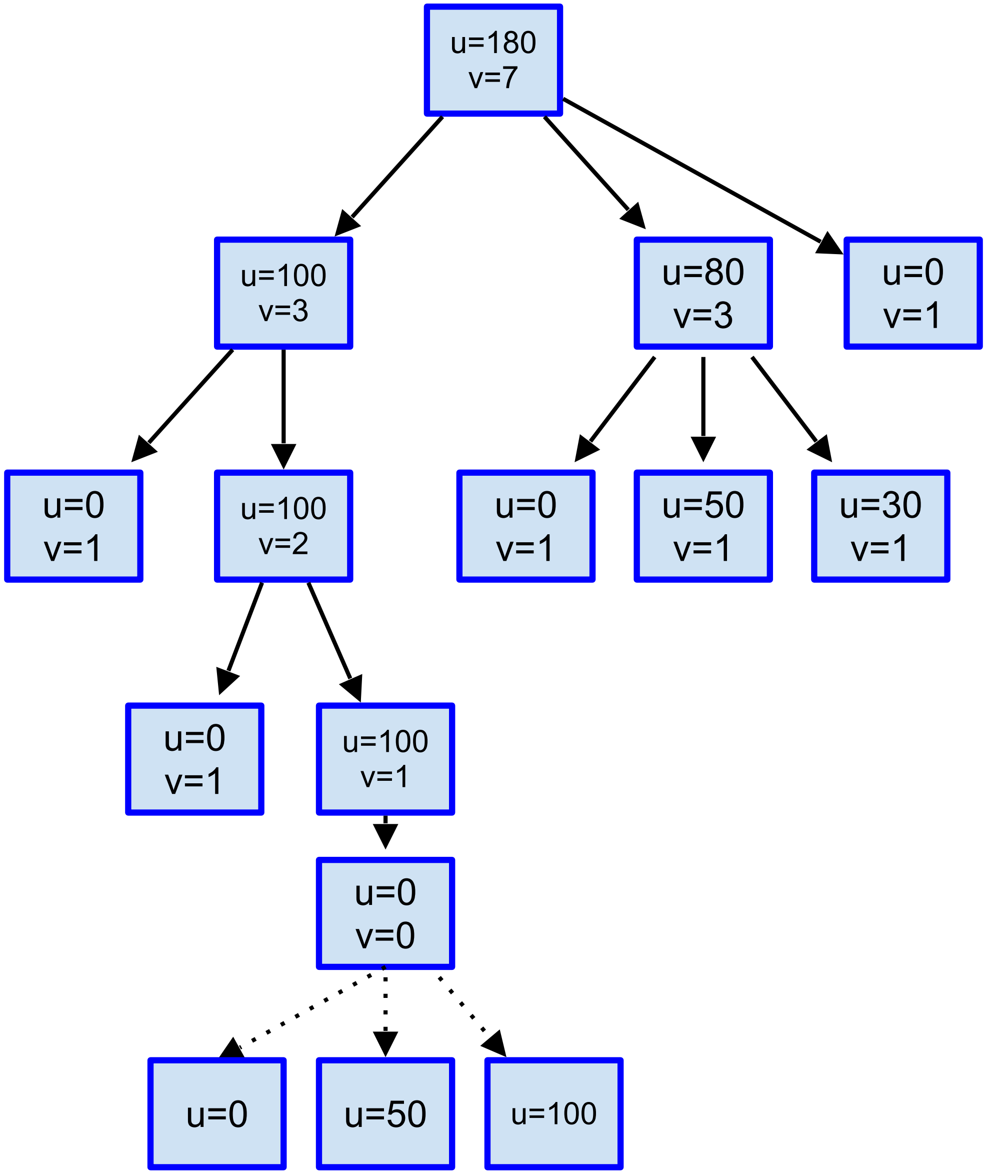
u=80
v=3
u=0
v=1
u=0
v=1
u=100
v=2
u=0
v=1
u=50
v=1
u=30
v=1
u=0
v=1
u=100
v=1
u=0
v=0
u=0

Simulate

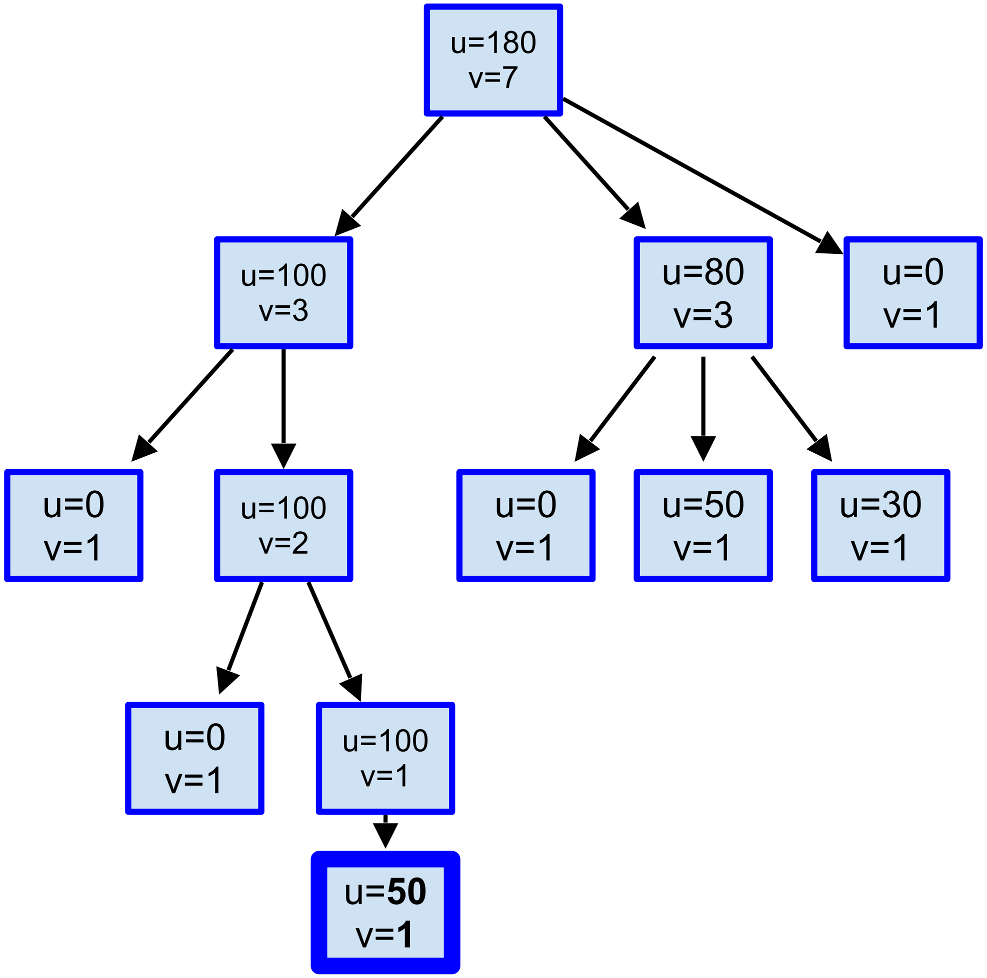


u=180 v=7
u=100 v=3
u=80 v=3
u=0 v=1
u=0 v=1
u=100 v=2
u=0 v=1
u=50 v=1
u=30 v=1
u=0 v=1
u=100 v=1
u=0 v=0
u=0
u=50

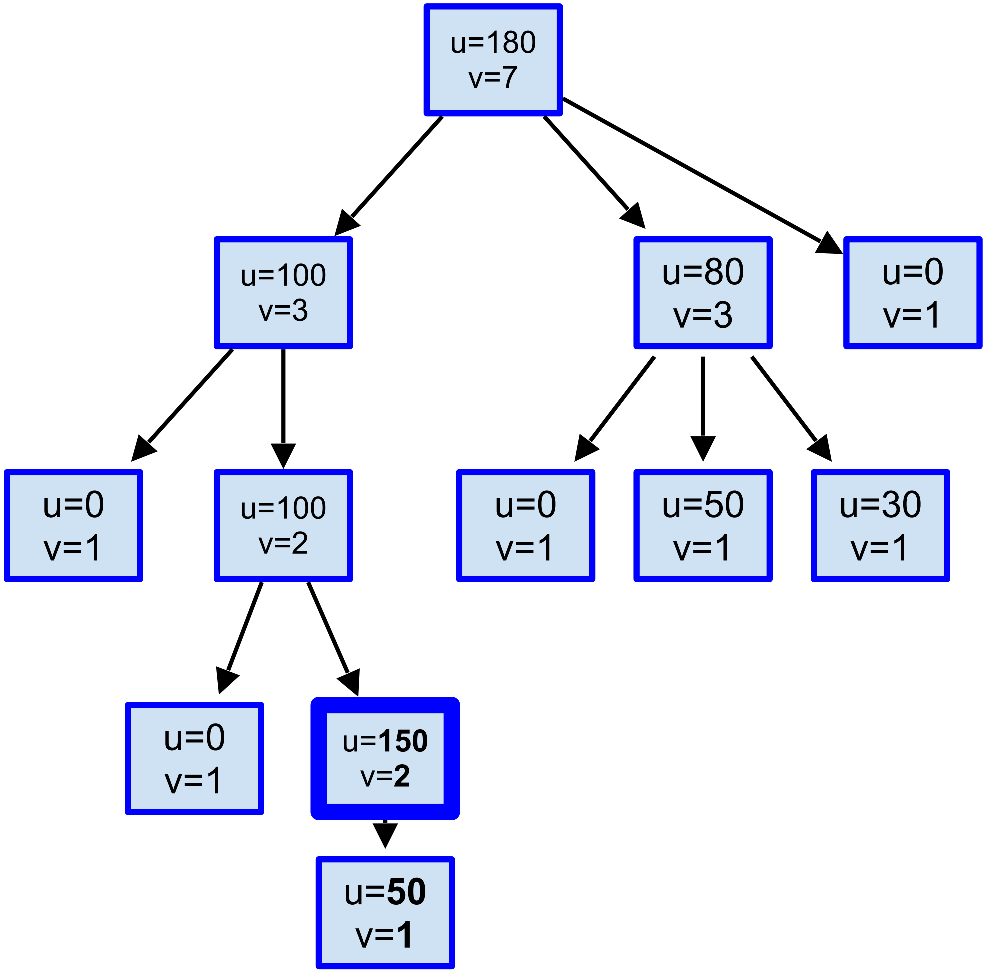
Simulate


u=180
v=7
u=100
v=3
u=80
v=3
u=0
v=1
u=0
v=1
u=100
v=2
u=0
v=1
u=50
v=1
u=30
v=1
u=0
v=1
u=100
v=1
u=0
v=0
u=0
u=50
u=100

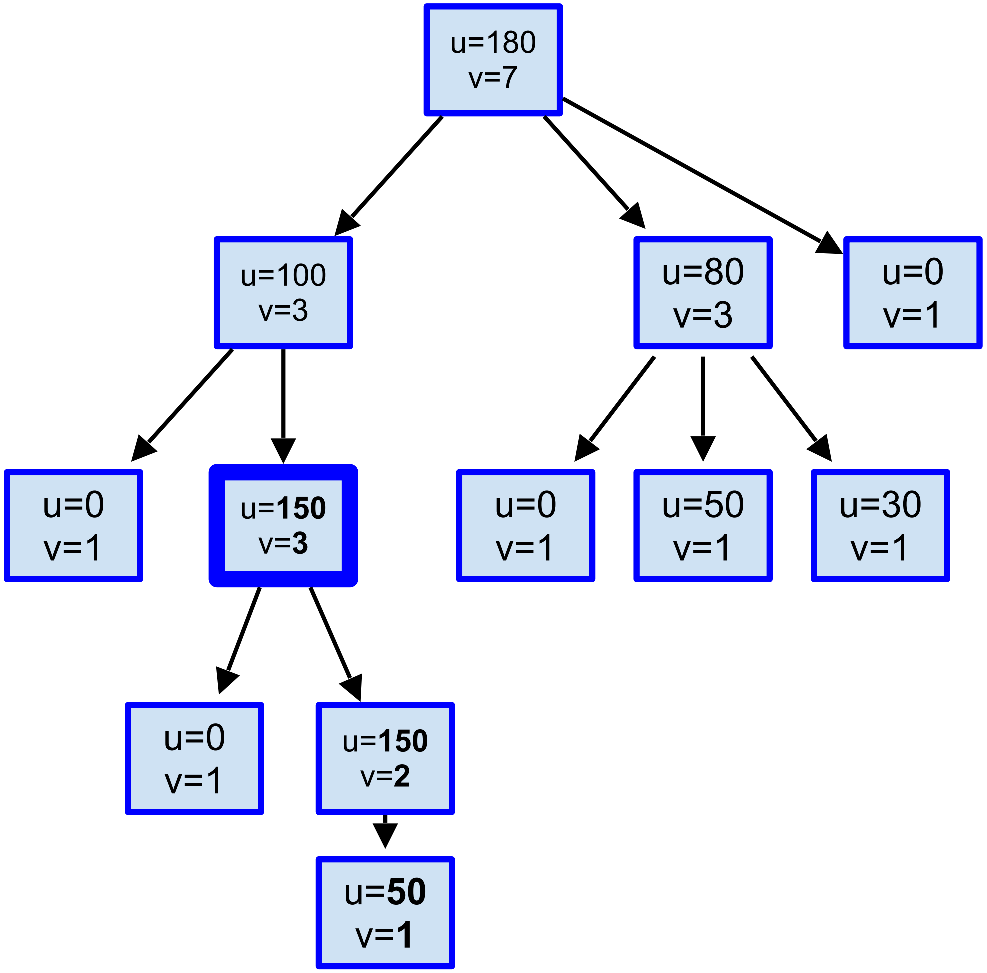
Backpropagate



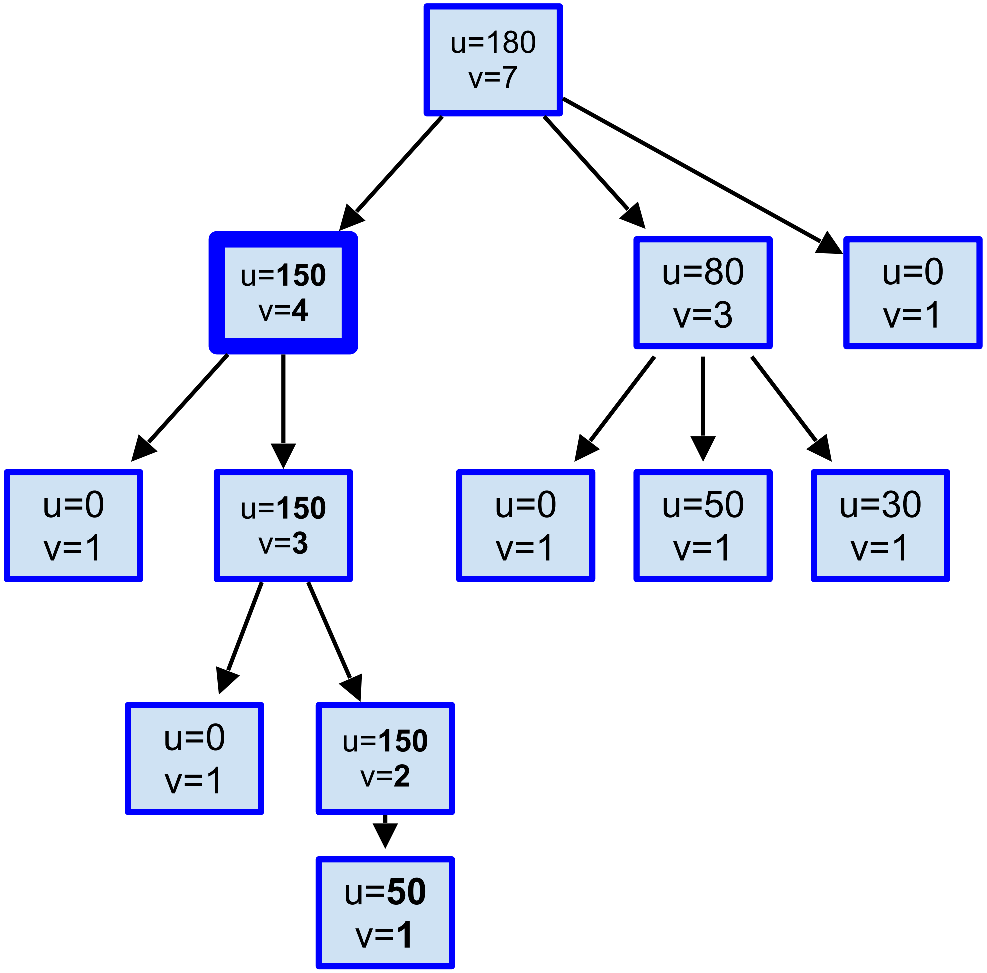
Backpropagate



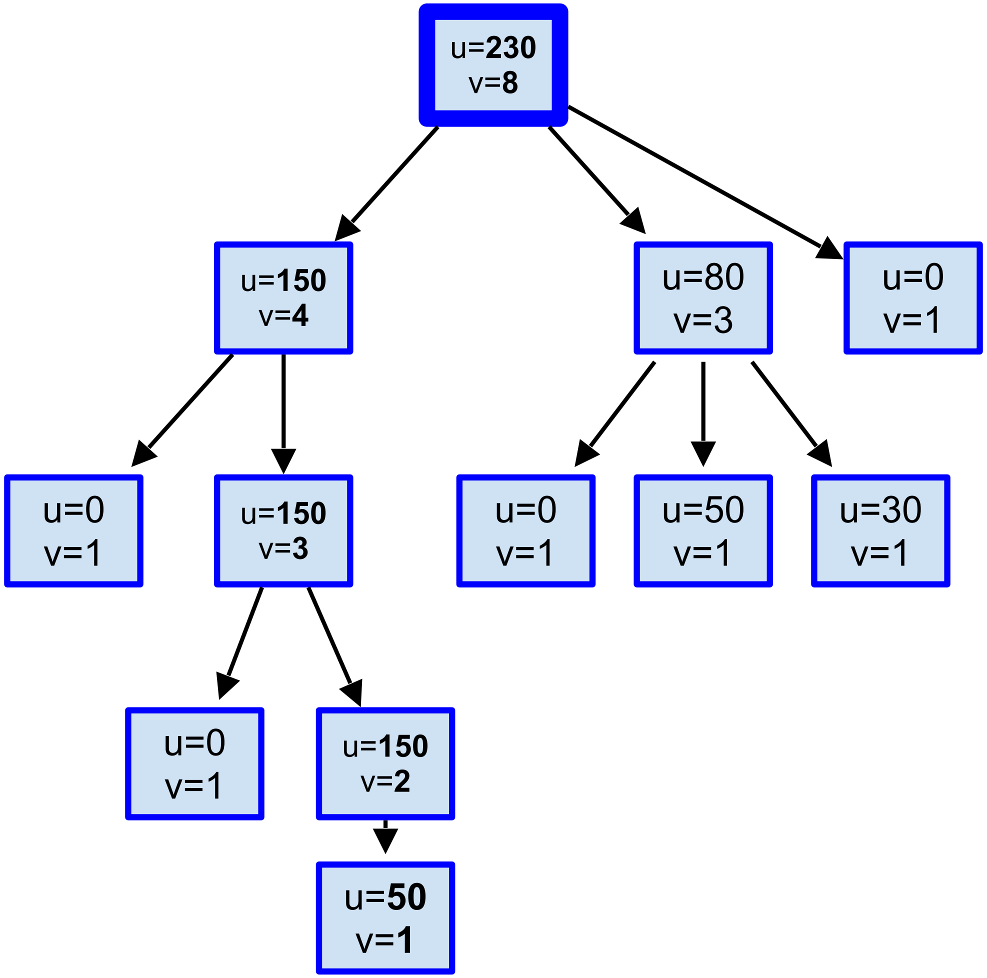
Backpropagate



Backpropagate



Backpropagate


u=**230**
v=**8**

u=**150**
v=**4**

u=80
v=3

u=0
v=1

u=0
v=1

u=**150**
v=**3**

u=0
v=1

u=50
v=1

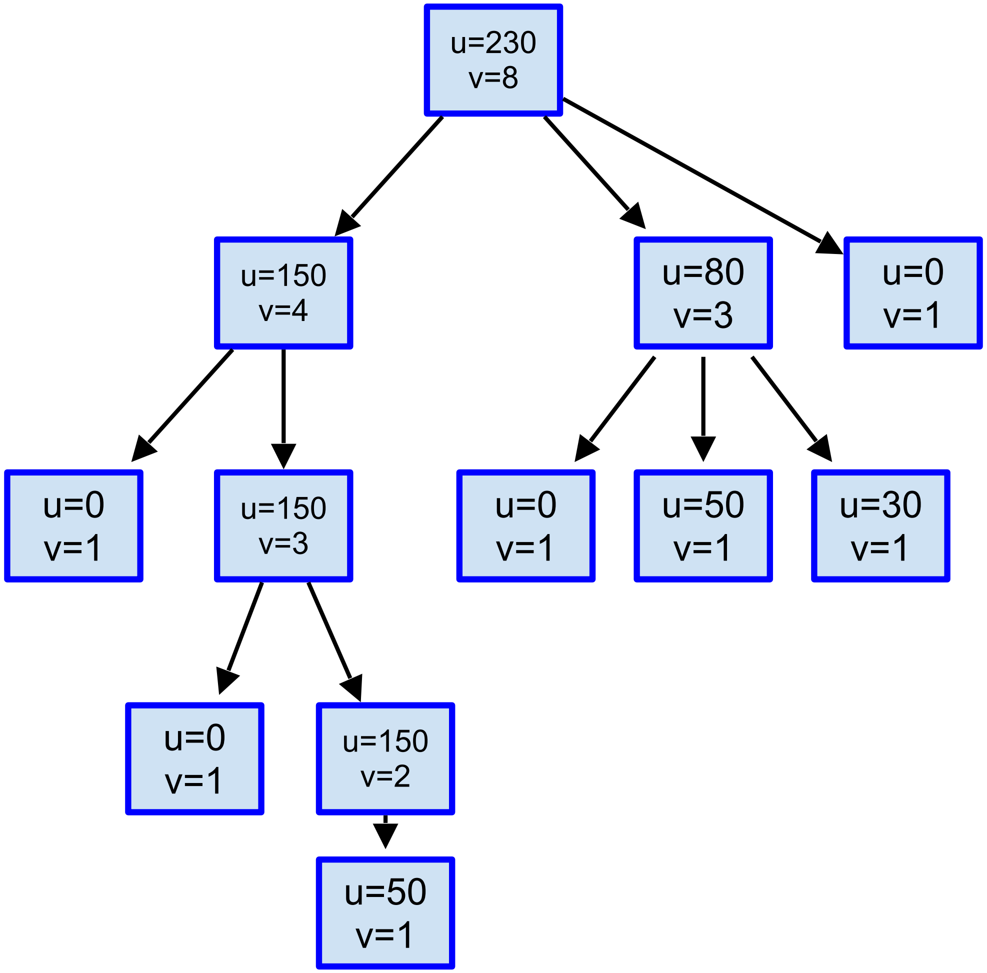
u=30
v=1

u=0
v=1

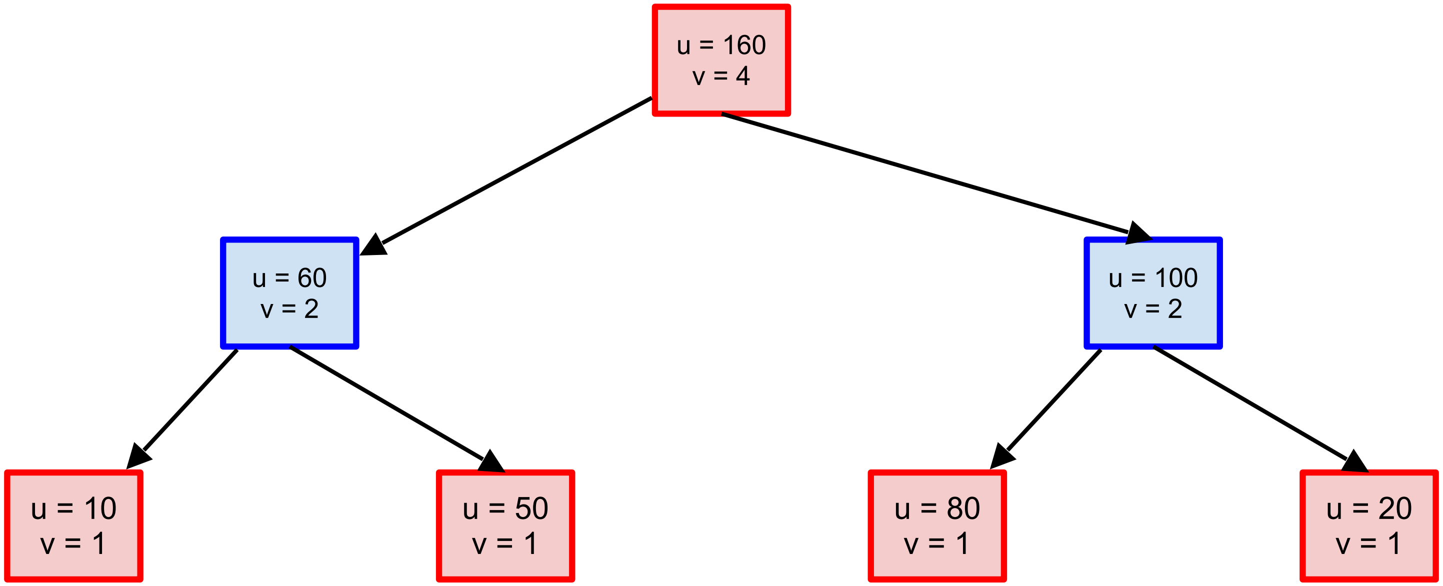
u=**150**
v=**2**

u=**50**
v=**1**

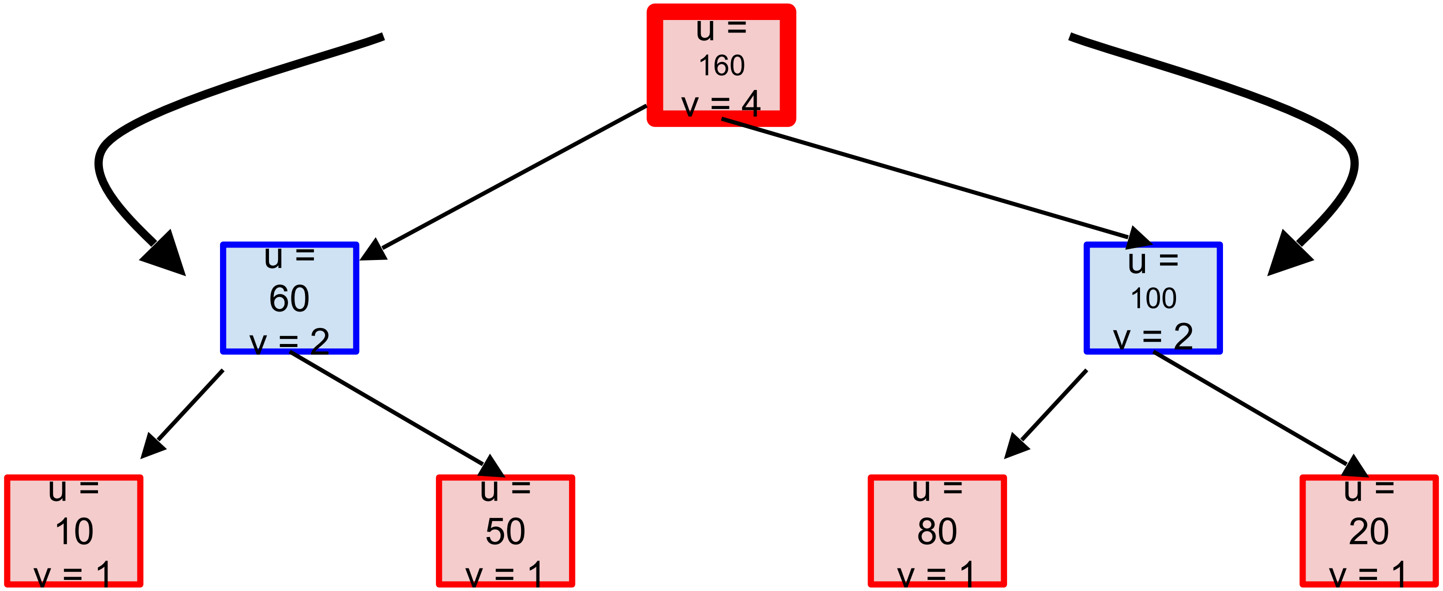
Backpropagate



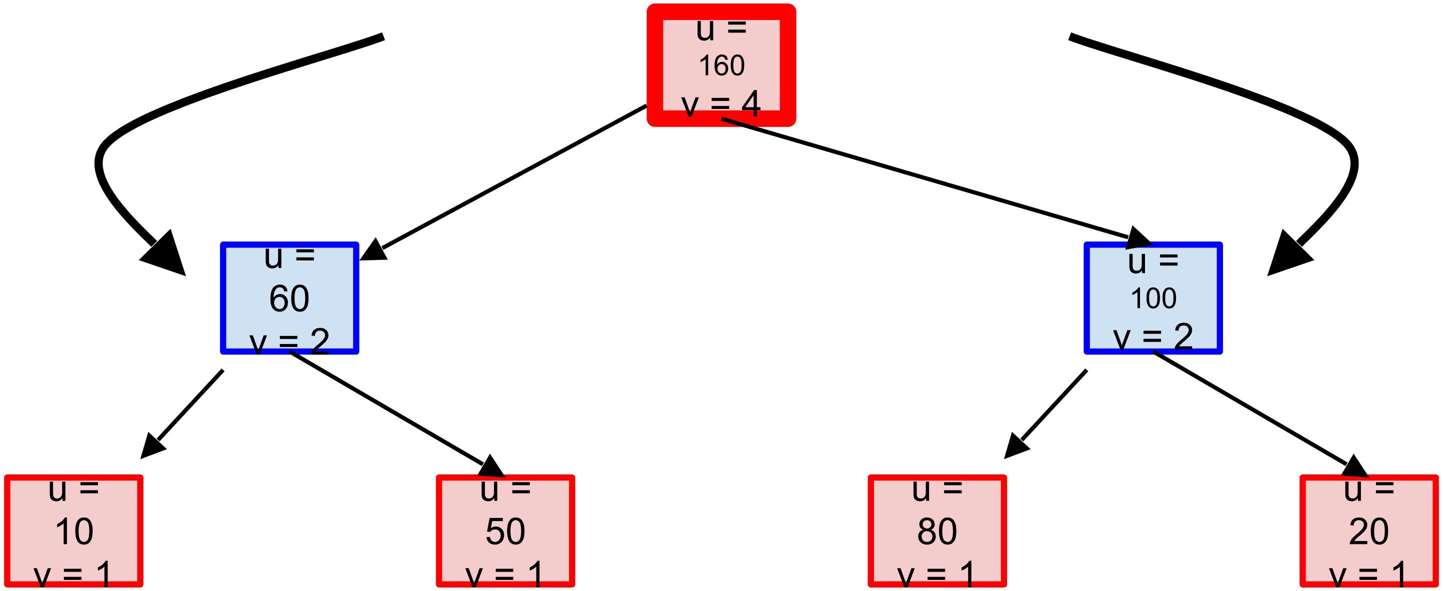
Multiple Players



Multiple Players

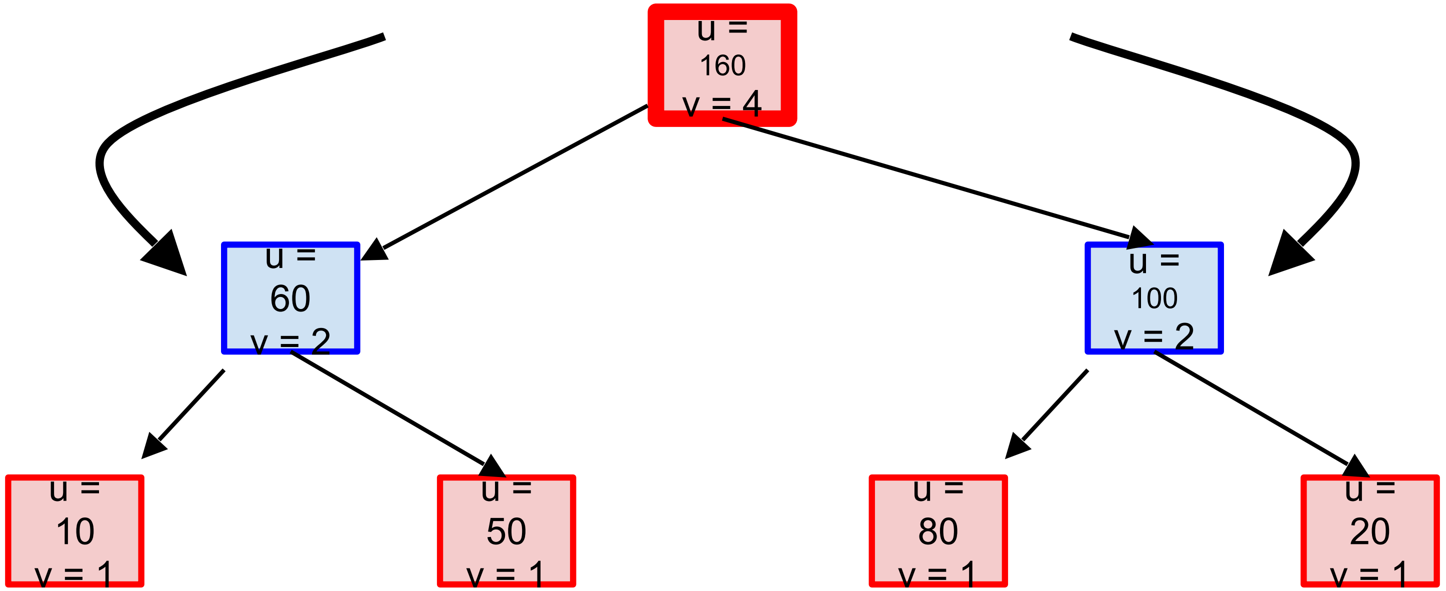


Multiple Players



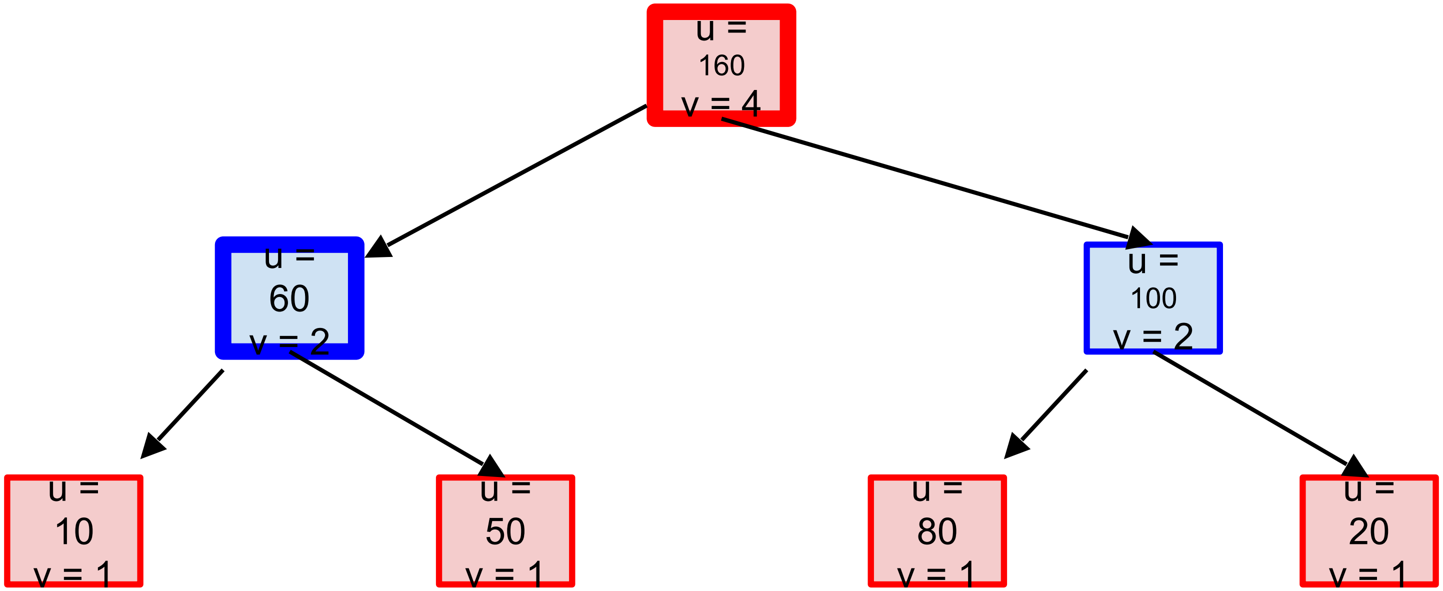
$$\text{selectValue} = \frac{\text{node.utility}}{\text{node.visits}} + C \times \sqrt{\frac{\ln(\text{node.parent.visits})}{\text{node.visits}}}$$

Multiple Players

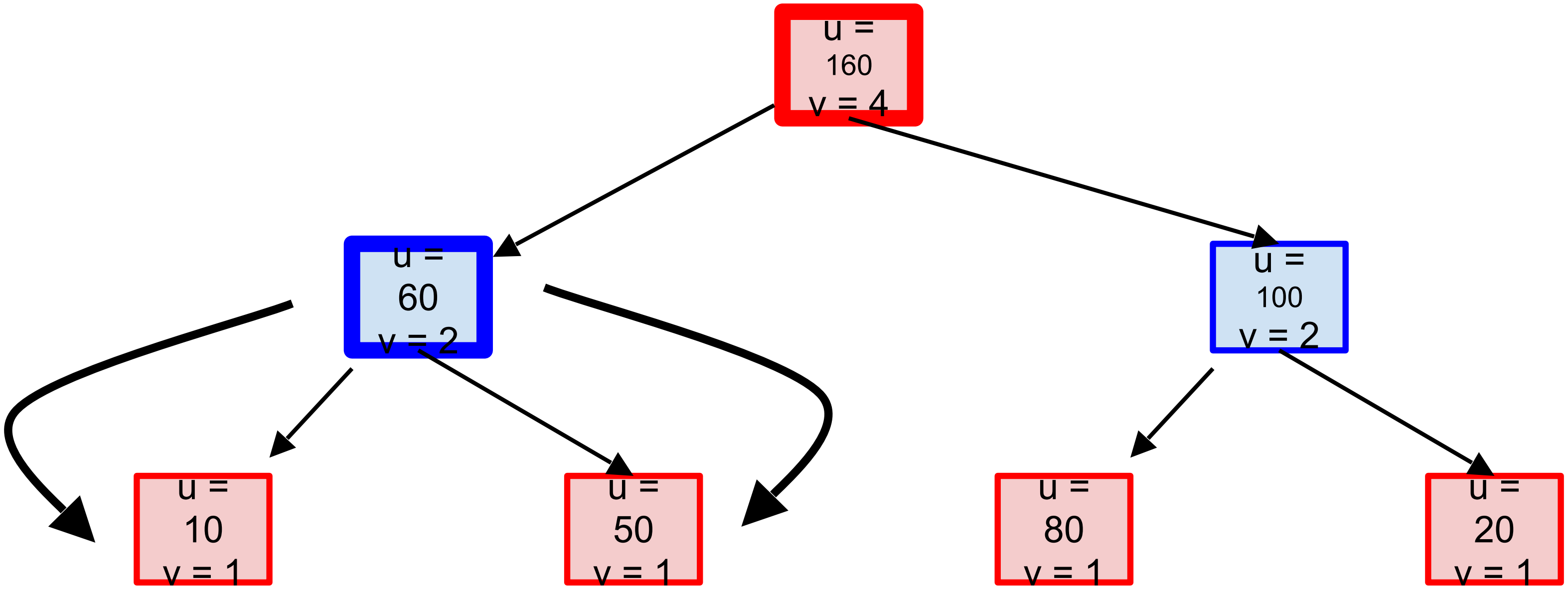


$$\text{selectValue} = \frac{\text{node.utility}}{\text{node.visits}} \overset{\overset{-1\times}{\vee}}{+} C \times \sqrt{\frac{\ln(\text{node.parent.visits})}{\text{node.visits}}}$$

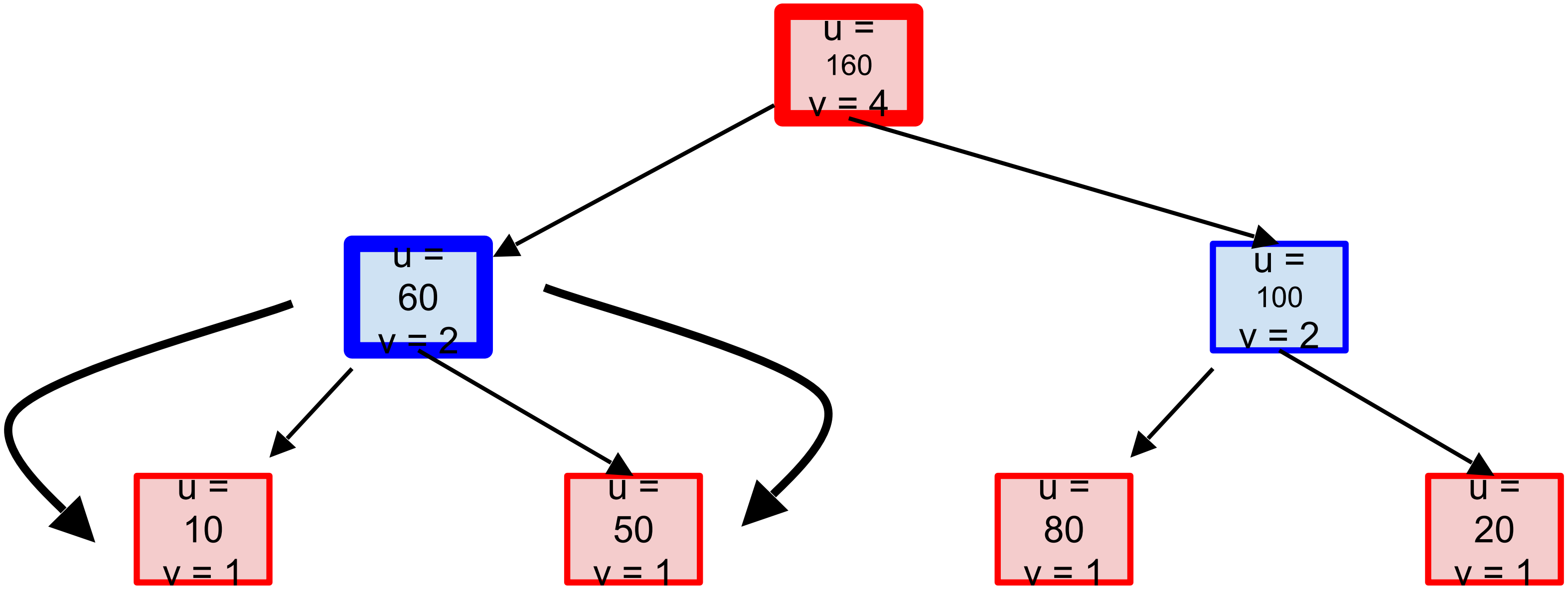
Multiple Players



Multiple Players

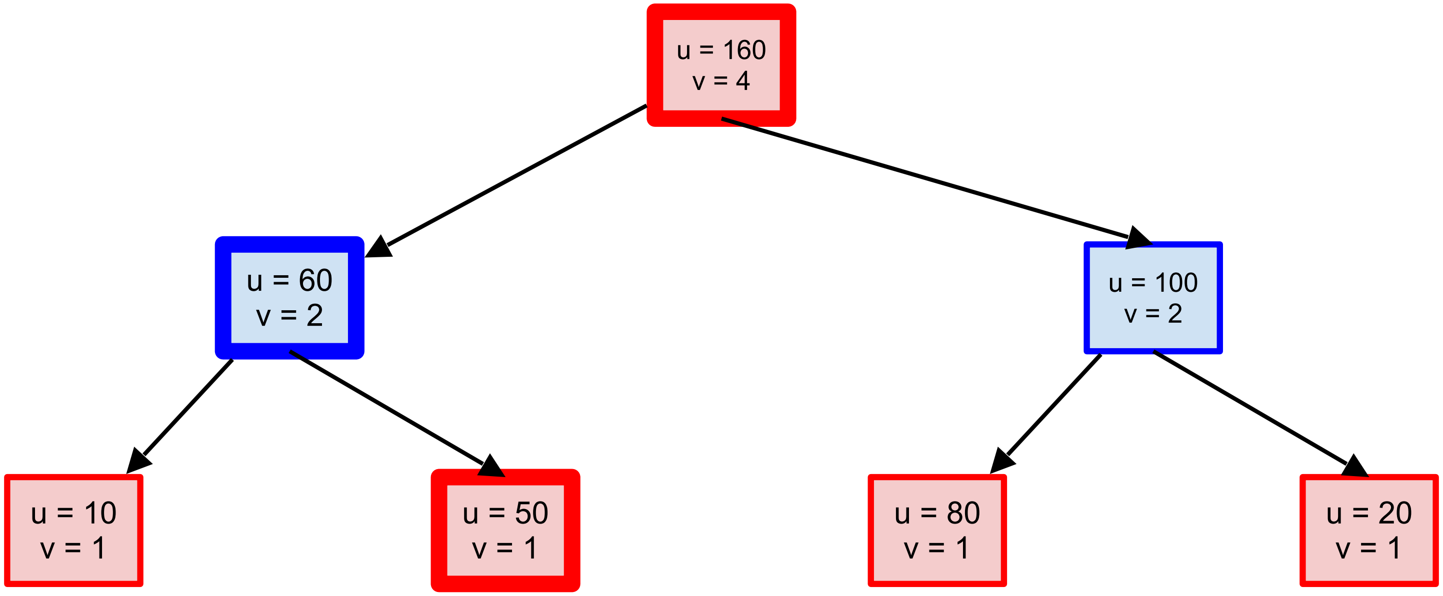


Multiple Players



$$\text{selectValue} = \frac{\text{node.utility}}{\text{node.visits}} + C \times \sqrt{\frac{\ln(\text{node.parent.visits})}{\text{node.visits}}}$$

Multiple Players



More Information

https://en.wikipedia.org/wiki/Monte_Carlo_tree_search

GENERAL GAME PLAYING

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